FIG. 1
Example Spectral Density Function

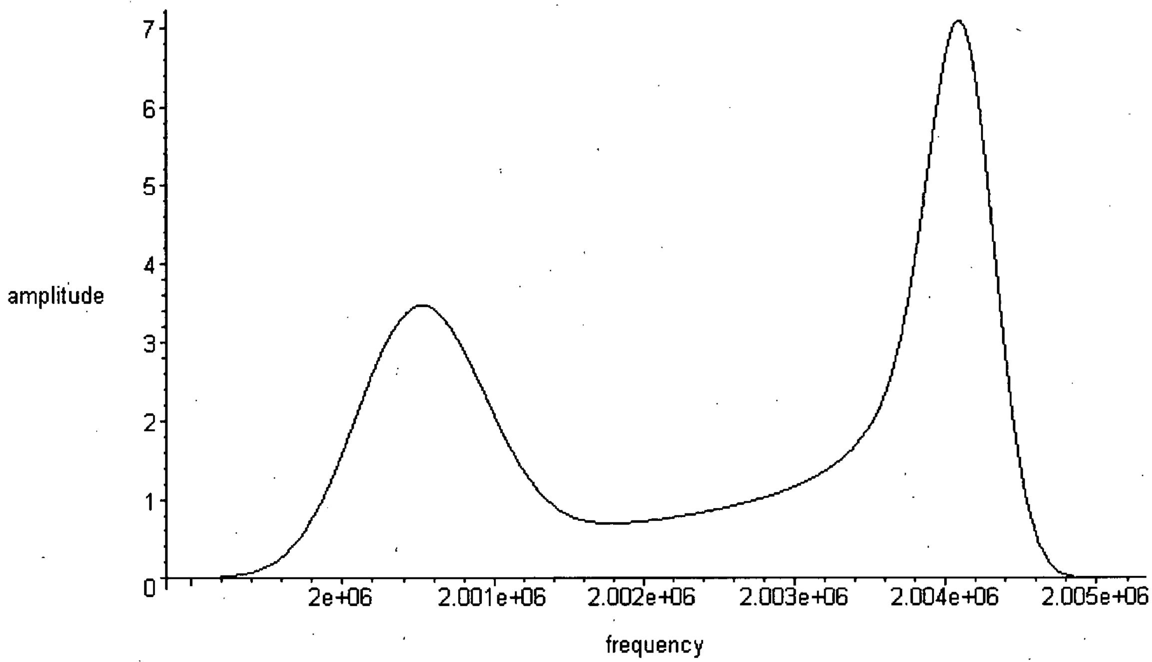

FIG. 2
Example Gaussian Pulse

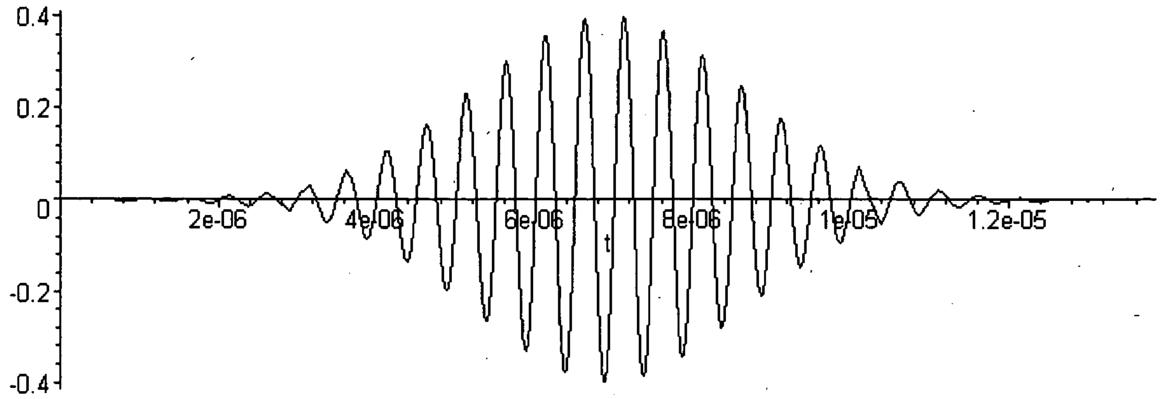

FIG. 3
Example Amplitude Function Made from 10 Piecewise Continuous Functions
(hollow dots are the knot points)

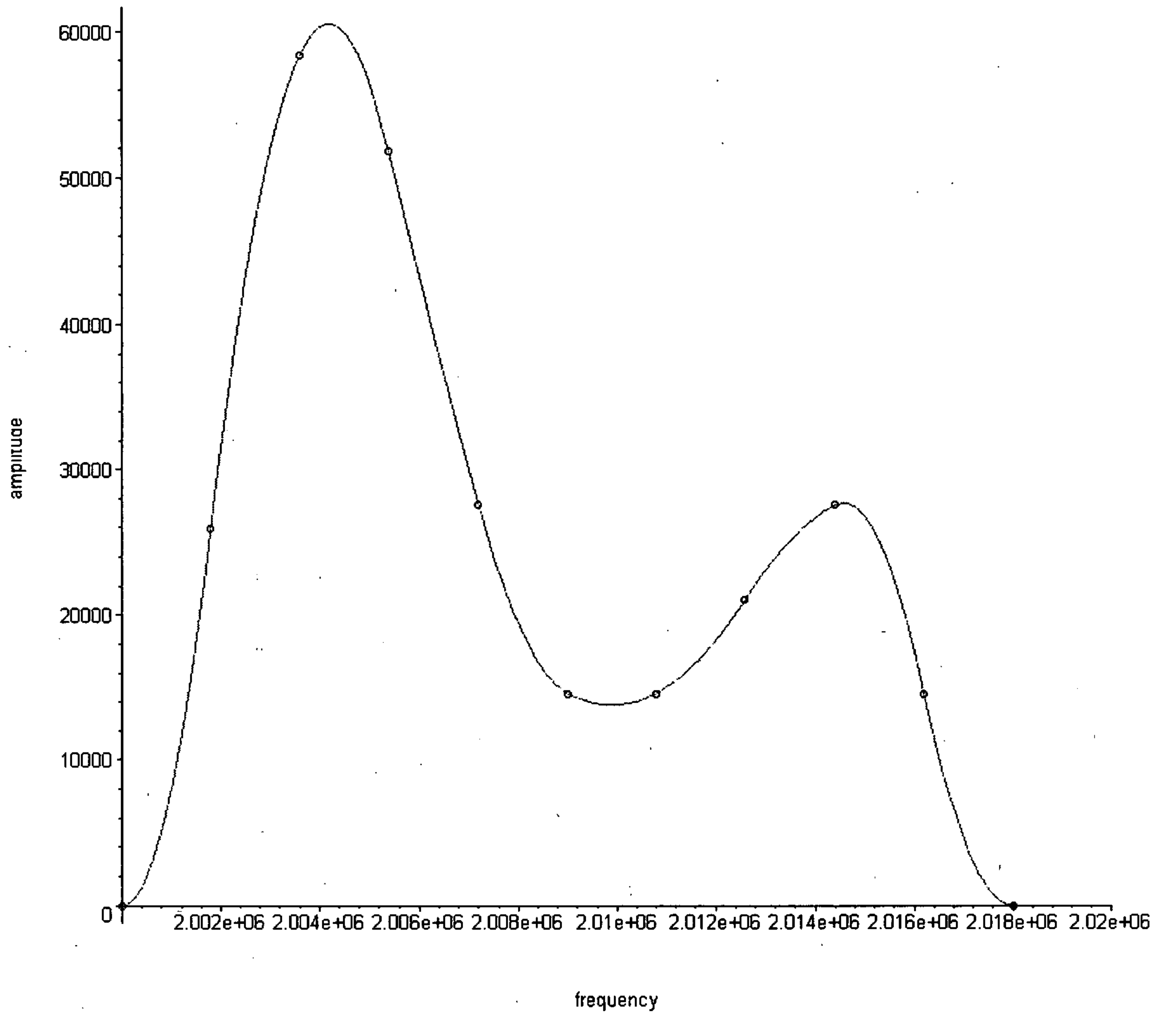

FIG. 4

Example Amplitude Function Composed of Multiple Functions with Overlapping Domains

(the top curve is the sum of the curves beneath it)

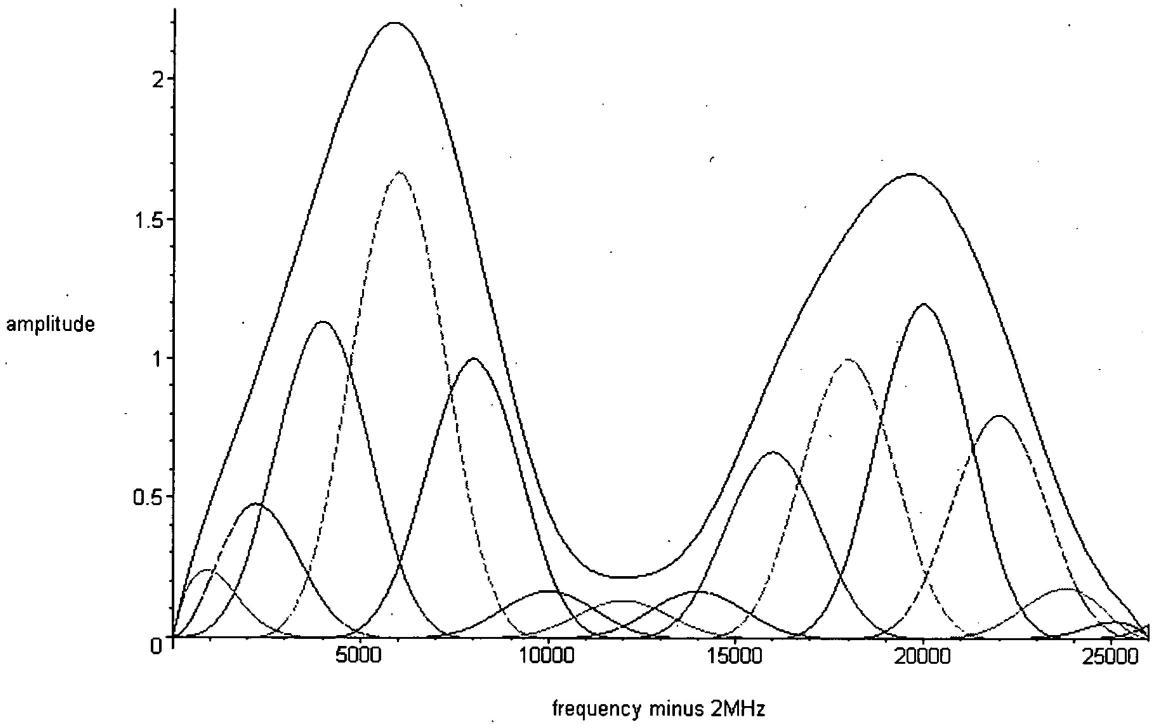

Figure 5
Example Phase Angle with Doppler

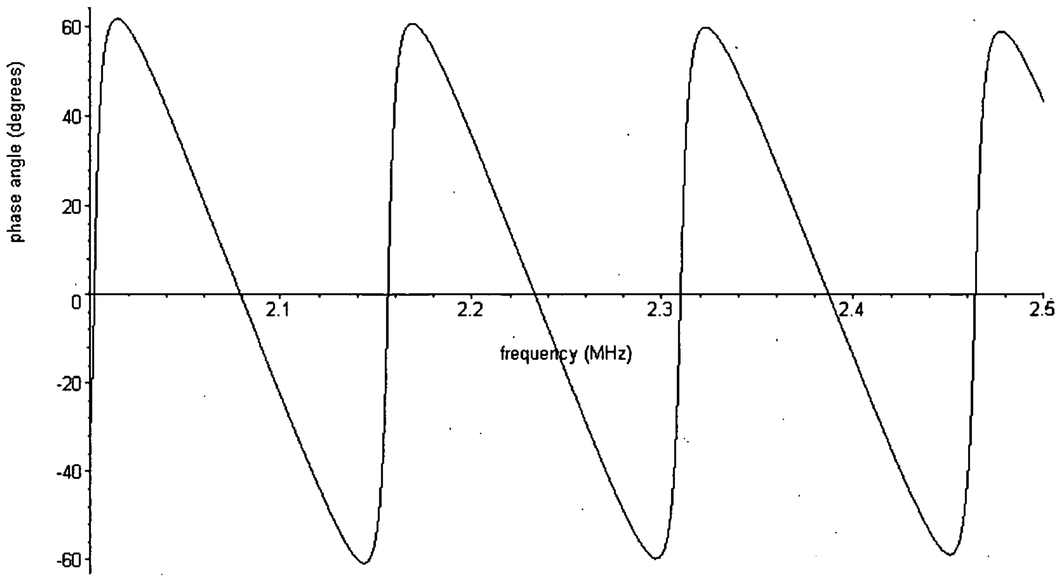

FIG. 6

Example Phase Angle with Doppler when Emitted Frequency Selected to Produce Approximately Linear Phase Angle Response

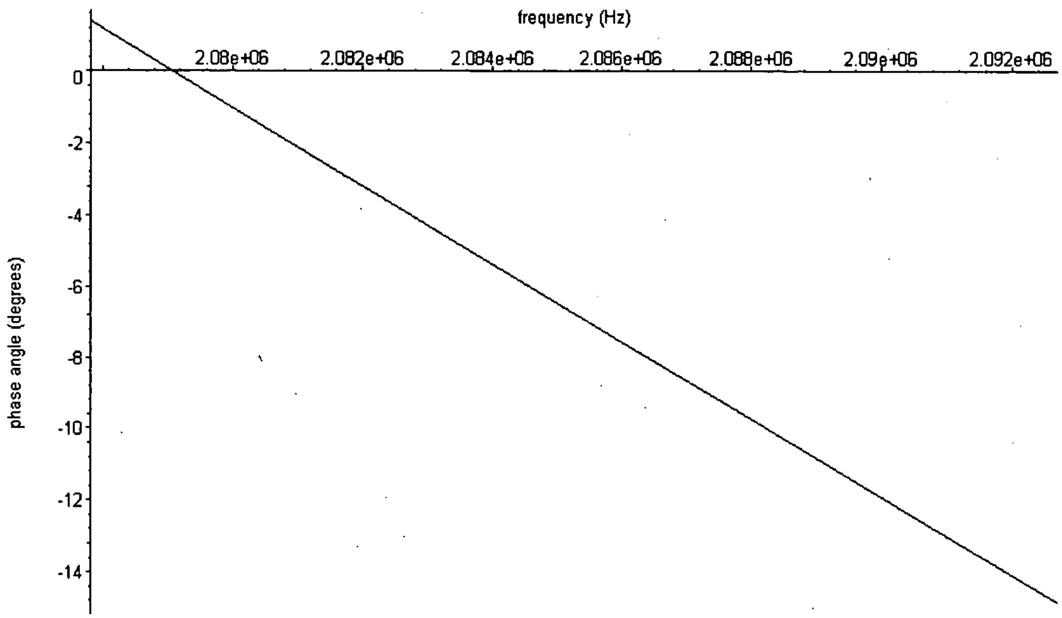

FIG. 7

Example Phase Angle with Doppler when Emitted Frequency Selected to Produce Always Positive and Approximately Linear Phase Angle Response

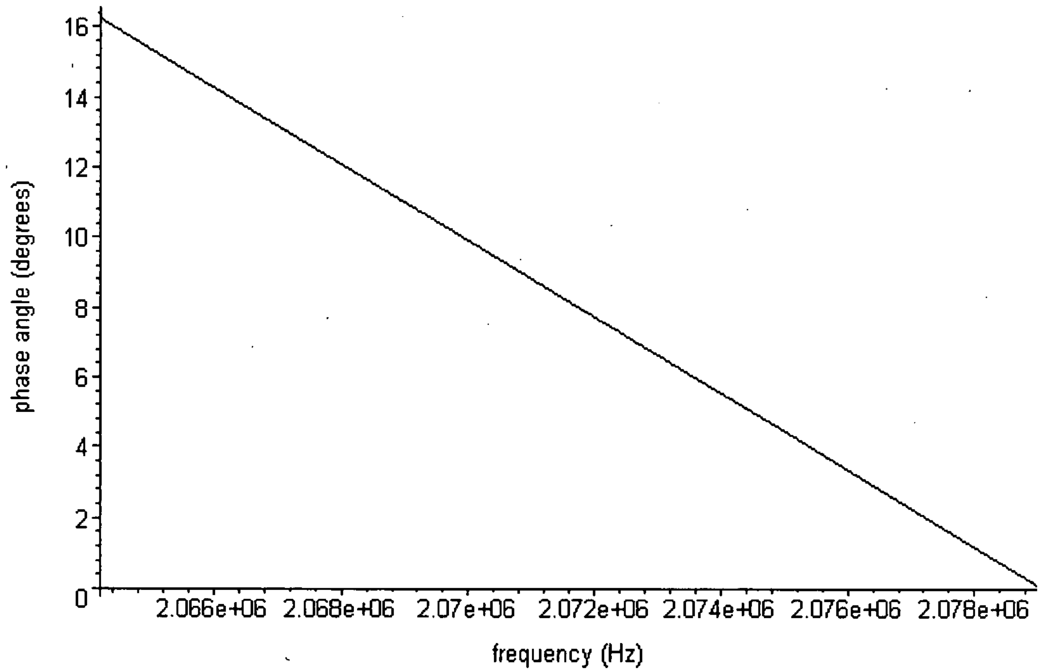

FIG. 8

Example sin(Phase Angle) with Doppler when Emitted Frequency Selected to Produce Positive Phase Angle Response Close to Zero Phase Angle

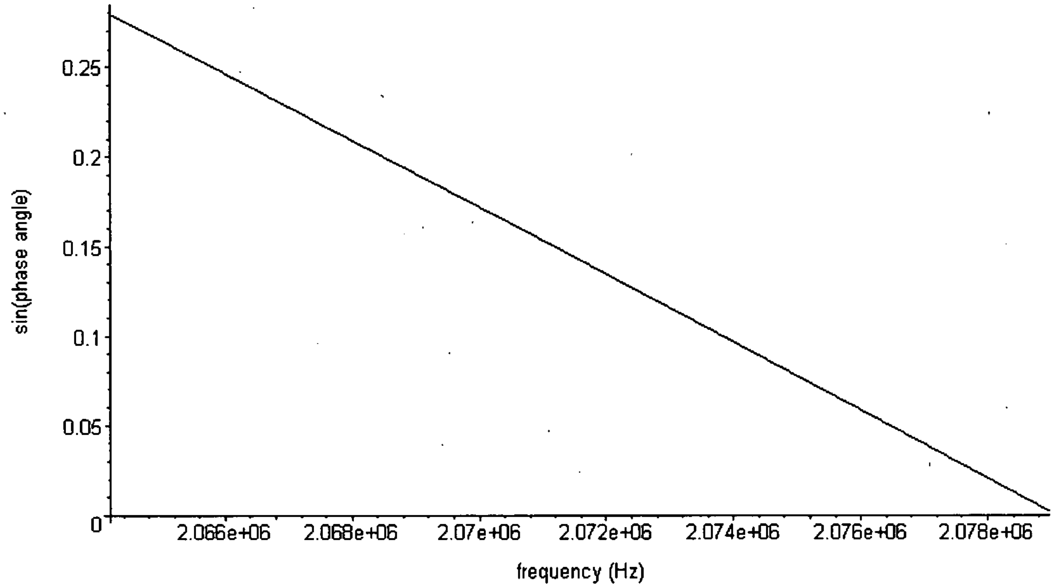

FIG. 9

Example cos(Phase Angle) with Doppler when Emitted Frequency Selected to Produce Positive Phase Angle Response Close to Zero Phase Angle

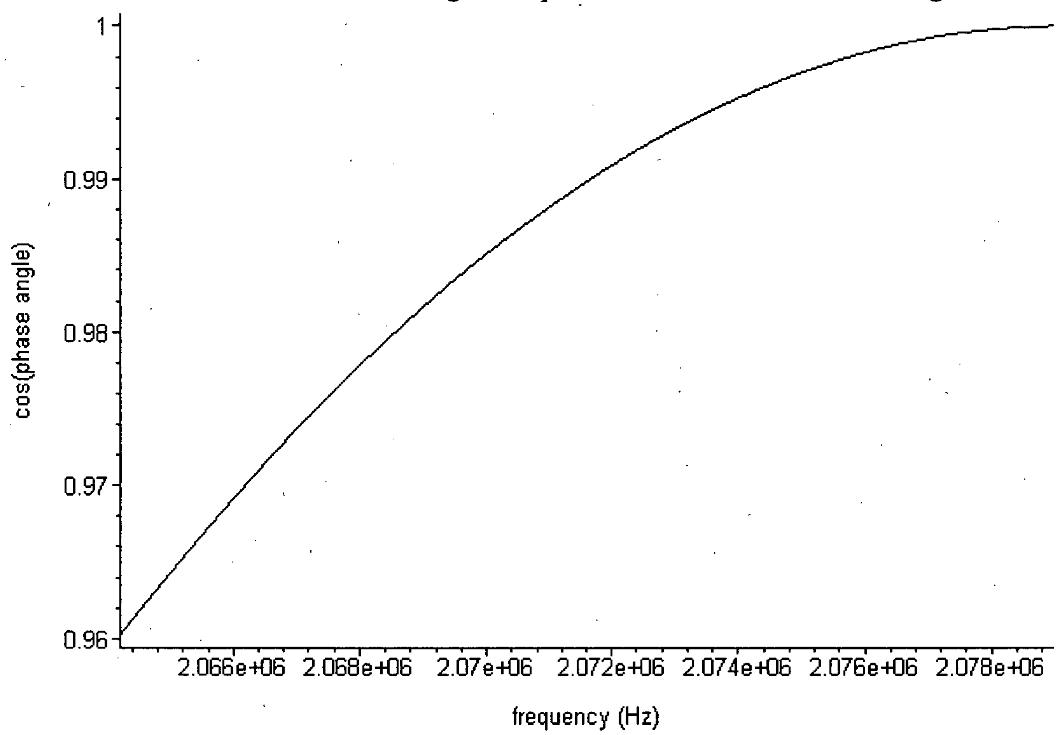

FIG. 10
Example Phase Angle with Doppler using 2 MHz Emitted Signal

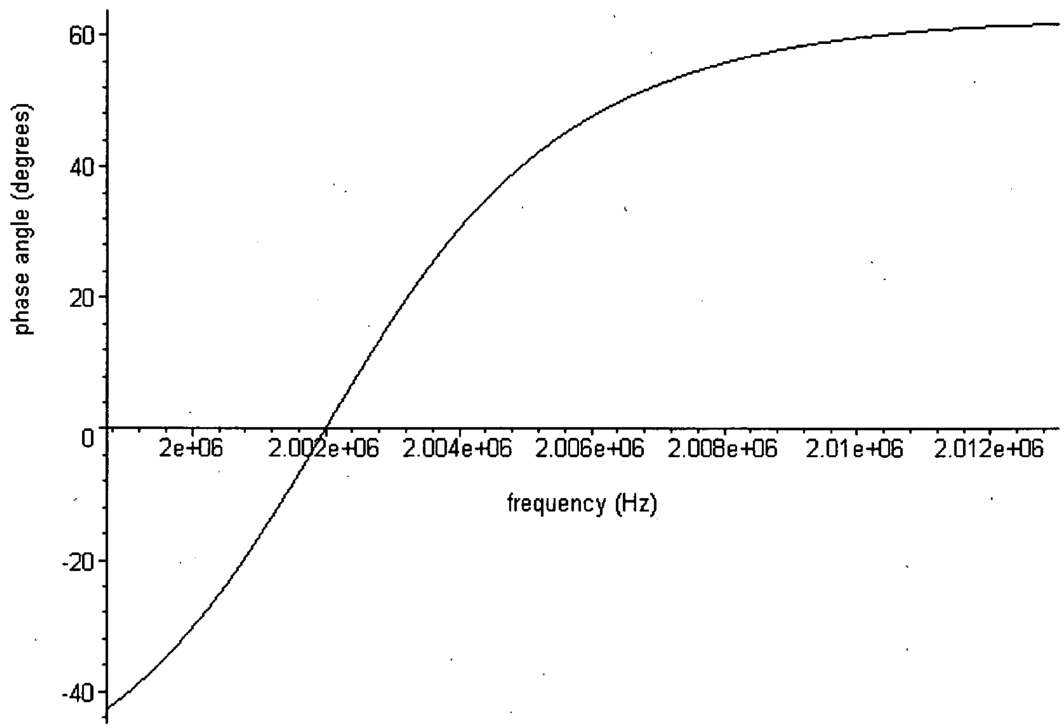

FIG. 11
Example cos(phase angle) with Doppler using 2 MHz Emitted Signal

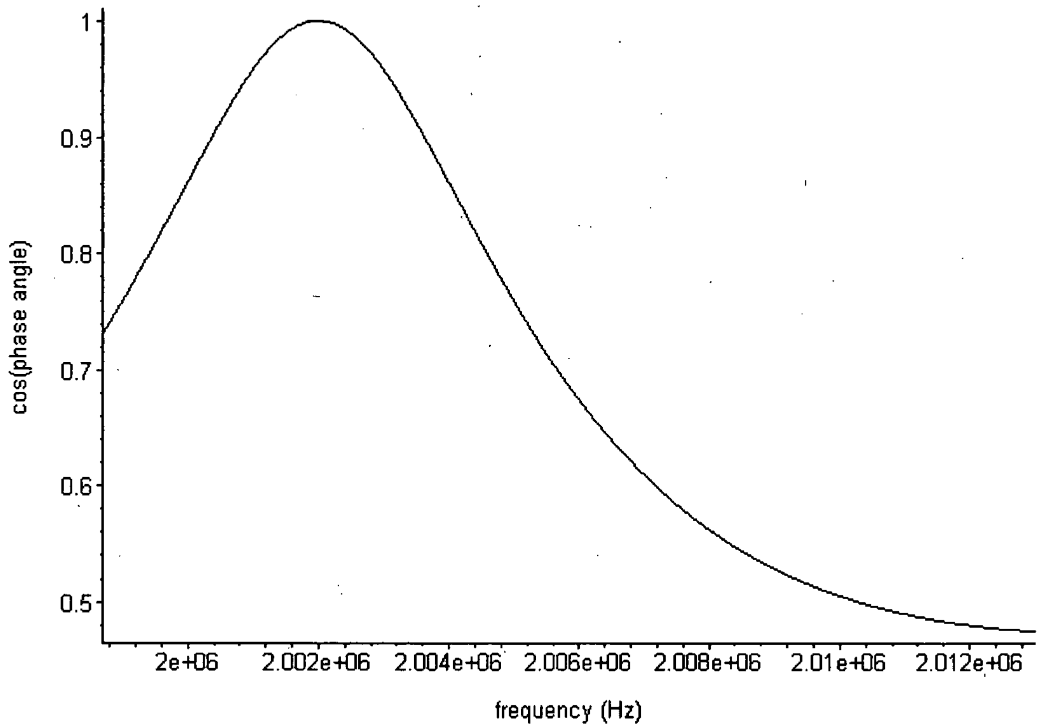

FIG. 12
Example sin(phase angle) with Doppler with 2 MHz Emitted Signal

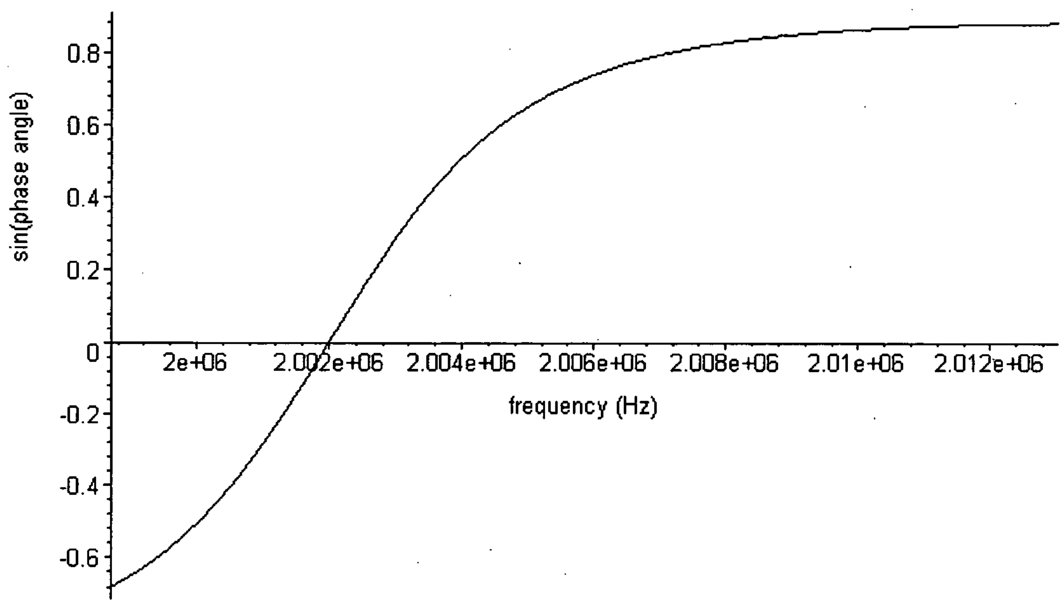

FIG. 13
Six Parameter Distribution for Velocity

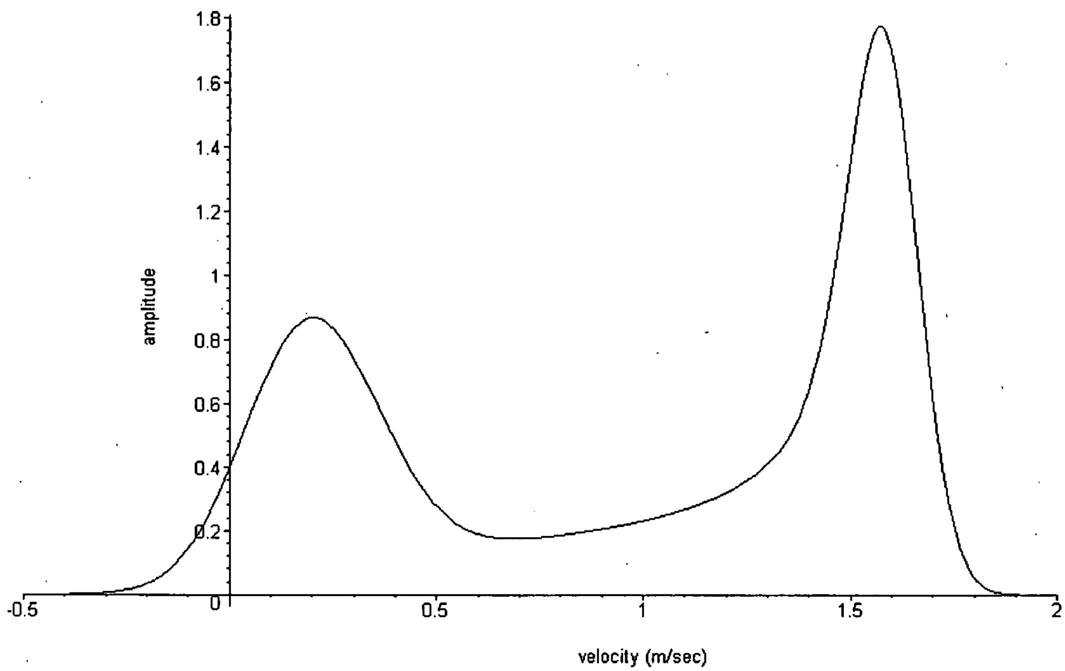

FIG. 14

Quadratic Function Segment Expression for f(t) without Continuity

$$
\begin{aligned}
f(t) = {} & \left(\sum_{j=1}^{N} \left(\left(\frac{1}{2} \frac{-\sin(2\pi t L_j) L_j^2 + \sin(2\pi t H_j) H_j^2}{\pi t} \right.\right.\right. \\
& \left. - \frac{1}{4} \frac{\sin(2\pi t H_j) - \sin(2\pi t L_j)}{\pi^3 t^3} + \frac{\frac{1}{2}\left(-\cos(2\pi t L_j) L_j + \cos(2\pi t H_j) H_j\right)}{\pi^2 t^2} \right) x_j \\
& + \left(\frac{1}{2} \frac{-\sin(2\pi t L_j) L_j + \sin(2\pi t H_j) H_j}{\pi t} + \frac{\frac{1}{4}\left(\cos(2\pi t H_j) - \cos(2\pi t L_j)\right)}{\pi^2 t^2} \right) y_j \\
& \left.\left. + \frac{\frac{1}{2}\left(\sin(2\pi t H_j) - \sin(2\pi t L_j)\right) z_j}{\pi t} \right) \right) + \left(\sum_{j=1}^{N} \left(\left(\frac{1}{2} \frac{\cos(2\pi t L_j) L_j^2 - \cos(2\pi t H_j) H_j^2}{\pi t} - \frac{1}{4} \frac{-\cos(2\pi t H_j) + \cos(2\pi t L_j)}{\pi^3 t^3} \right.\right.\right. \\
& \left. - \frac{1}{2} \frac{\sin(2\pi t L_j) L_j - \sin(2\pi t H_j) H_j}{\pi^2 t^2} \right) a_j \\
& \left.\left. + \left(\frac{1}{2} \frac{\cos(2\pi t L_j) L_j - \cos(2\pi t H_j) H_j}{\pi t} - \frac{1}{4} \frac{-\sin(2\pi t H_j) + \sin(2\pi t L_j)}{\pi^2 t^2} \right) b_j + \frac{\frac{1}{2}\left(-\cos(2\pi t H_j) + \cos(2\pi t L_j)\right) c_j}{\pi t} \right) \right)
\end{aligned}
$$

FIG. 15

Parameters for Example

Number of function segments: N = 7

M = 20

Lowest frequency in basis: $H_0 = 0.1996961309 \quad 10^7$

Starting time: $t_0 = 0.00006843506494$

Frequency increment: $H_\delta = 2207.908286$

Ending time: $t_M = 0.00007493194556$

Figure 16

Example ||A|| Matrix

$$
\begin{bmatrix}
0.8203\,10^{10} & 0.6331\,10^{10} & 0.3611\,10^{10} & 0.1218\,10^{10} & 0.5626\,10^{8} & -0.4482\,10^{10} & -0.5209\,10^{10} & -0.4925\,10^{10} & -0.3361\,10^{10} & -0.1319\,10^{10} \\
0.1346\,10^{10} & 0.3307\,10^{10} & 0.4440\,10^{10} & 0.3724\,10^{10} & 0.1696\,10^{10} & 0.1309\,10^{11} & 0.1109\,10^{11} & 0.7411\,10^{10} & 0.3434\,10^{10} & 0.7954\,10^{9} \\
-0.1704\,10^{11} & -0.1562\,10^{11} & -0.1164\,10^{11} & -0.6280\,10^{10} & -0.1893\,10^{10} & -0.5179\,10^{10} & -0.1706\,10^{10} & 0.1658\,10^{10} & 0.2869\,10^{10} & 0.1717\,10^{10} \\
0.1481\,10^{11} & 0.9966\,10^{10} & 0.3906\,10^{10} & -0.2836\,10^{9} & -0.1120\,10^{10} & -0.1785\,10^{11} & -0.1792\,10^{11} & -0.1488\,10^{11} & -0.9045\,10^{10} & -0.3161\,10^{10} \\
0.1343\,10^{11} & 0.1588\,10^{11} & 0.1535\,10^{11} & 0.1065\,10^{11} & 0.4226\,10^{10} & 0.2574\,10^{11} & 0.2027\,10^{11} & 0.1179\,10^{11} & 0.4082\,10^{10} & 0.2339\,10^{9} \\
-0.3484\,10^{11} & -0.3000\,10^{11} & -0.2036\,10^{11} & -0.9561\,10^{10} & -0.2240\,10^{10} & 0.2920\,10^{10} & 0.8181\,10^{10} & 0.1159\,10^{11} & 0.9996\,10^{10} & 0.4623\,10^{10} \\
0.1249\,10^{11} & 0.4762\,10^{10} & -0.3243\,10^{10} & -0.6434\,10^{10} & -0.3971\,10^{10} & -0.3852\,10^{11} & -0.3579\,10^{11} & -0.2708\,10^{11} & -0.1479\,10^{11} & -0.4493\,10^{10} \\
0.3418\,10^{11} & 0.3478\,10^{11} & 0.2935\,10^{11} & 0.1809\,10^{11} & 0.6373\,10^{10} & 0.2942\,10^{11} & 0.2041\,10^{11} & 0.8411\,10^{10} & -0.2369\,10^{9} & -0.2168\,10^{10} \\
-0.4323\,10^{11} & -0.3471\,10^{11} & -0.2061\,10^{11} & -0.7312\,10^{10} & -0.4937\,10^{9} & 0.2158\,10^{11} & 0.2593\,10^{11} & 0.2562\,10^{11} & 0.1809\,10^{11} & 0.7252\,10^{10} \\
-0.3285\,10^{10} & -0.1078\,10^{11} & -0.1618\,10^{11} & -0.1437\,10^{11} & -0.6751\,10^{10} & -0.4967\,10^{11} & -0.4344\,10^{11} & -0.2996\,10^{11} & -0.1425\,10^{11} & -0.3377\,10^{10} \\
0.4665\,10^{11} & 0.4393\,10^{11} & 0.3375\,10^{11} & 0.1866\,10^{11} & 0.5713\,10^{10} & 0.1602\,10^{11} & 0.6875\,10^{10} & -0.3284\,10^{10} & -0.7717\,10^{10} & -0.4921\,10^{10} \\
-0.3126\,10^{11} & -0.2234\,10^{11} & -0.9637\,10^{10} & -0.9902\,10^{8} & 0.2245\,10^{10} & 0.3510\,10^{11} & 0.3615\,10^{11} & 0.3102\,10^{11} & 0.1938\,10^{11} & 0.6886\,10^{10} \\
-0.1860\,10^{11} & -0.2265\,10^{11} & -0.2290\,10^{11} & -0.1645\,10^{11} & -0.6670\,10^{10} & -0.3885\,10^{11} & -0.3181\,10^{11} & -0.1926\,10^{11} & -0.7002\,10^{10} & -0.5391\,10^{9} \\
0.3792\,10^{11} & 0.3368\,10^{11} & 0.2360\,10^{11} & 0.1138\,10^{11} & 0.2727\,10^{10} & -0.1891\,10^{10} & -0.7552\,10^{10} & -0.1208\,10^{11} & -0.1106\,10^{11} & -0.5284\,10^{10} \\
-0.1094\,10^{11} & -0.5185\,10^{10} & 0.1692\,10^{10} & 0.4949\,10^{10} & 0.3267\,10^{10} & 0.3027\,10^{11} & 0.2888\,10^{11} & 0.2253\,10^{11} & 0.1261\,10^{11} & 0.3892\,10^{10} \\
-0.1932\,10^{11} & -0.2012\,10^{11} & -0.1755\,10^{11} & -0.1112\,10^{11} & -0.3987\,10^{10} & -0.1775\,10^{11} & -0.1306\,10^{11} & -0.5880\,10^{10} & -0.2589\,10^{9} & 0.1244\,10^{10} \\
0.1869\,10^{11} & 0.1560\,10^{11} & 0.9637\,10^{10} & 0.3585\,10^{10} & 0.3077\,10^{9} & -0.8601\,10^{10} & -0.1060\,10^{11} & -0.1096\,10^{11} & -0.8017\,10^{10} & -0.3287\,10^{10} \\
0.5433\,10^{9} & 0.2813\,10^{10} & 0.4824\,10^{10} & 0.4561\,10^{10} & 0.2216\,10^{10} & 0.1551\,10^{11} & 0.1398\,10^{11} & 0.9954\,10^{10} & 0.4866\,10^{10} & 0.1180\,10^{10} \\
-0.1053\,10^{11} & -0.1016\,10^{11} & -0.8053\,10^{10} & -0.4566\,10^{10} & -0.1421\,10^{10} & -0.3976\,10^{10} & -0.2056\,10^{10} & 0.4279\,10^{9} & 0.1697\,10^{10} & 0.1162\,10^{10} \\
0.5391\,10^{10} & 0.4081\,10^{10} & 0.1913\,10^{10} & 0.1417\,10^{9} & -0.3689\,10^{9} & -0.5704\,10^{10} & -0.5995\,10^{10} & -0.5321\,10^{10} & -0.3419\,10^{10} & -0.1237\,10^{10}
\end{bmatrix}
$$

FIG. 17

Example ||P|| Matrix

$$
\begin{bmatrix}
-0.1106 & -0.09701 & 0.03599 & 0.02232 & 0.08865 & -0.1066 & -0.01061 & 0.01016 & 0.04178 & 0.04867 & -0.1111 & 0.02669 & -0.01052 & 0.07140 & 0.003079 & -0.1052 & 0.05433 & -0.03459 & 0.1329 & -0.05895 \\
0.2056 & 0.8055 & -0.2468 & 0.1065 & -0.6869 & 0.4025 & 0.2762 & -0.06595 & -0.06227 & -0.5585 & 0.5727 & 0.03943 & 0.08169 & -0.3012 & -0.3731 & 0.6851 & -0.1258 & 0.2557 & -0.8206 & -0.1547 \\
0.6081 & -1.729 & 0.4506 & -0.6620 & 1.409 & -0.1543 & -0.8687 & 0.1087 & -0.2864 & 1.435 & -0.7179 & -0.4686 & -0.1750 & 0.2214 & 1.322 & -1.200 & -0.2094 & -0.5118 & 1.360 & 1.289 \\
-1.870 & 1.111 & -0.1783 & 1.043 & -0.8198 & -0.9007 & 0.9457 & -0.01928 & 0.7811 & -1.266 & -0.2567 & 0.8417 & 0.1200 & 0.4566 & -1.597 & 0.3979 & 0.8116 & 0.2872 & -0.3208 & -2.195 \\
1.661 & 0.3813 & -0.2324 & -0.6092 & -0.4113 & 1.248 & -0.2773 & -0.08587 & -0.6674 & 0.1068 & 1.030 & -0.5518 & 0.02606 & -0.7798 & 0.6253 & 0.7160 & -0.7770 & 0.1579 & -0.9697 & 1.389 \\
0.07551 & -0.1193 & 0.02930 & -0.05976 & 0.09443 & 0.01232 & -0.06378 & 0.007381 & -0.03401 & 0.1060 & -0.03256 & -0.04460 & -0.01117 & 0.0001873 & 0.1078 & -0.07319 & -0.02996 & -0.03331 & 0.07848 & 0.1205 \\
-0.6860 & 0.1697 & 0.0001696 & 0.3251 & -0.09710 & -0.4093 & 0.2499 & 0.005766 & 0.2822 & -0.2805 & -0.2368 & 0.2750 & 0.01423 & 0.2376 & -0.4413 & -0.04652 & 0.3074 & 0.02590 & 0.1145 & -0.7028 \\
1.559 & 0.8468 & -0.3553 & -0.4500 & -0.8051 & 1.333 & -0.06138 & -0.1083 & -0.6130 & -0.2800 & 1.259 & -0.4489 & 0.08594 & -0.8711 & 0.2850 & 1.067 & -0.7480 & 0.3130 & -1.384 & 1.084 \\
-1.125 & -2.253 & 0.7237 & -0.05713 & 1.952 & -1.500 & -0.6310 & 0.1987 & 0.4118 & 1.427 & -1.873 & 0.08831 & -0.2248 & 1.082 & 0.7490 & -2.058 & 0.6072 & -0.7255 & 2.514 & -0.1087 \\
-0.1935 & 1.889 & -0.5196 & 0.5198 & -1.569 & 0.4897 & 0.8269 & -0.1272 & 0.1101 & -1.458 & 1.019 & 0.3411 & 0.1933 & -0.4497 & -1.198 & 1.438 & 0.01034 & 0.5740 & -1.683 & -0.9506
\end{bmatrix}
$$

FIG. 18
Example Envelope Function

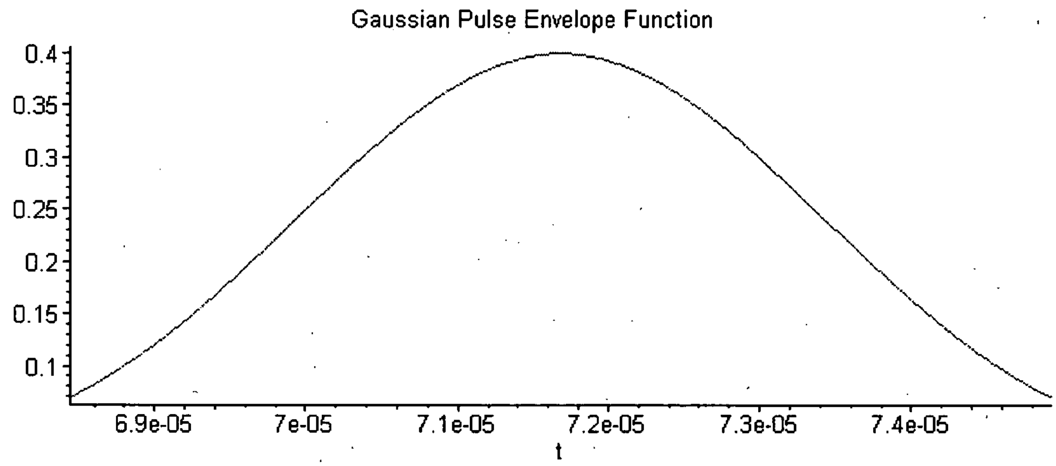

FIG. 19
Spectral Density Component Functions Used to Calculate Example f(t) Values

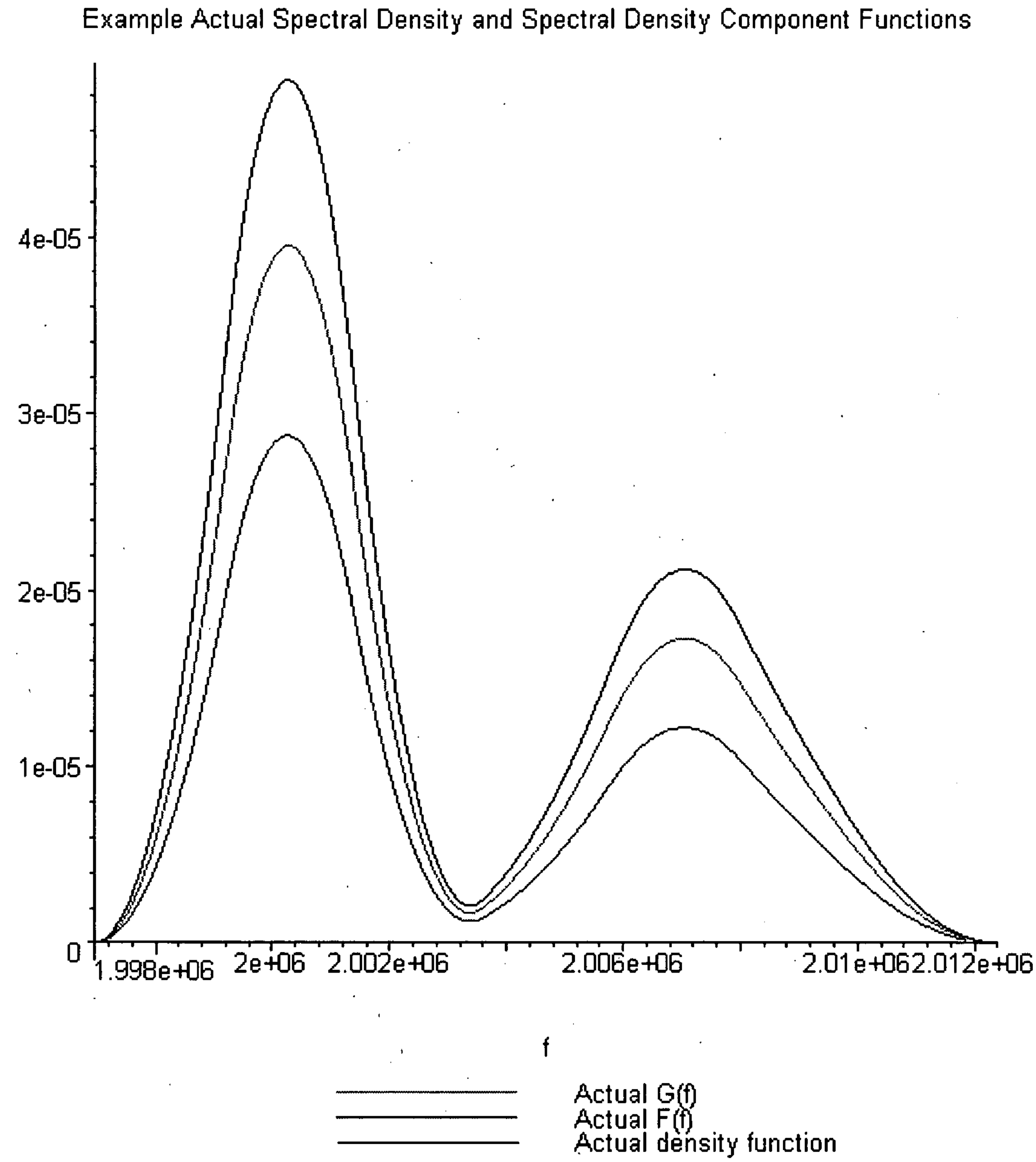

FIG. 20
Example Signal and Locations of Measurement Points

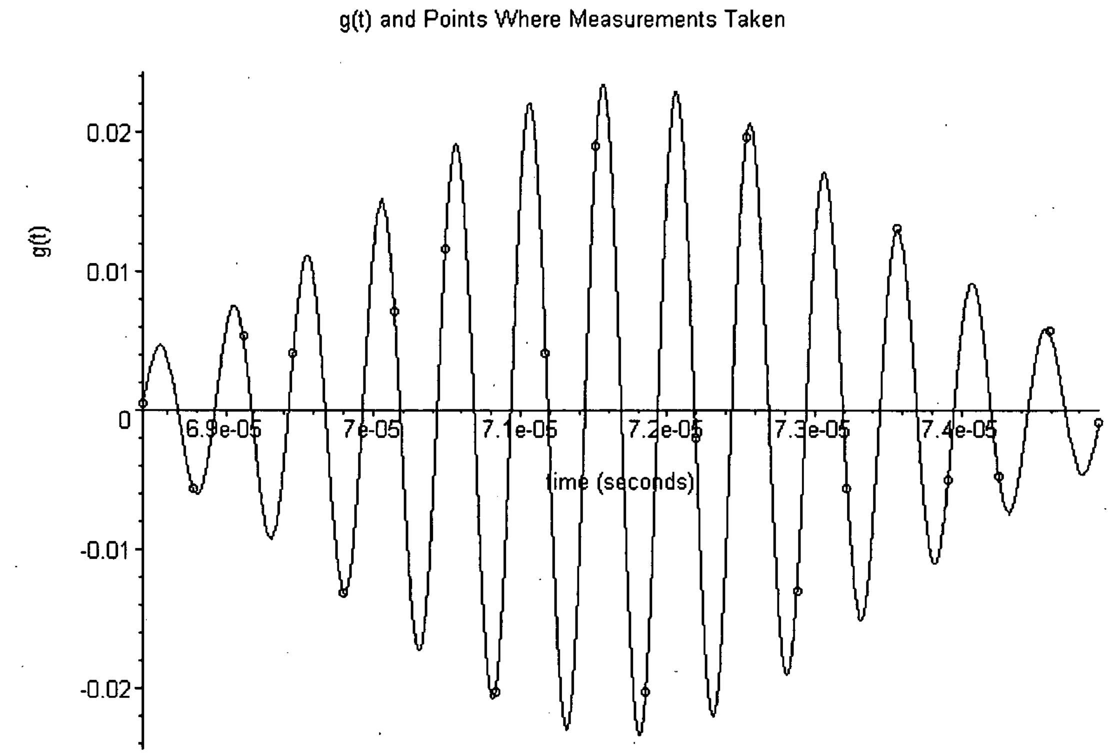

FIG. 21
Example Signal after Adjust for Gaussian Envelope and Locations of Measurement Points
(The period of this signal is much longer than the short segment shown.)

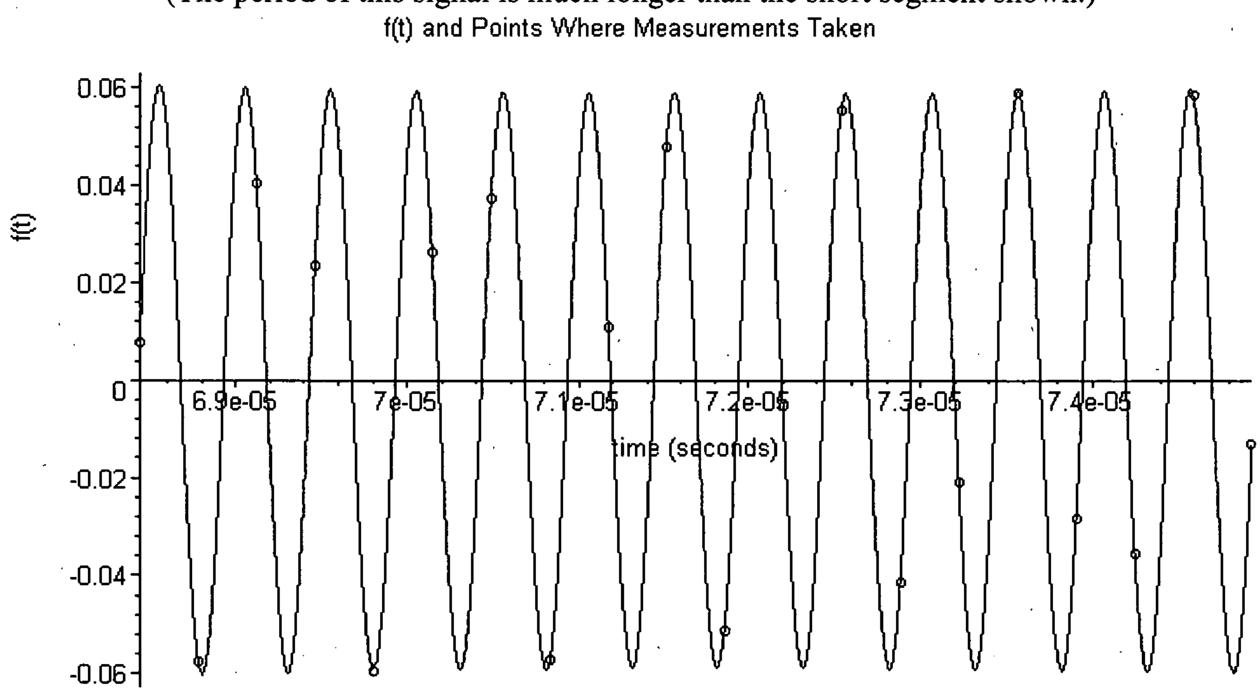

FIG. 22
Example Calculated Results for G(f) and Actual G(f)
(The calculated result overlays the actual G(f).)

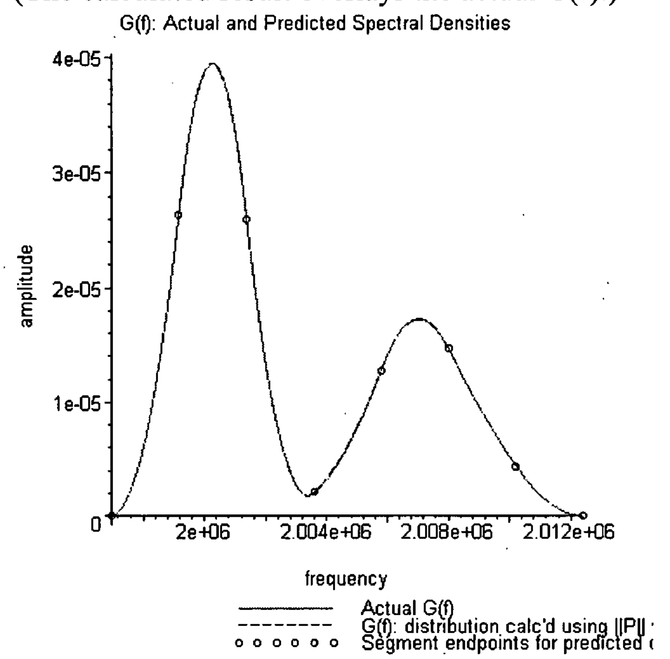

FIG. 23
Example Calculated Results for F(f) and Actual F(f)
(The calculated result overlays the actual F(f).)

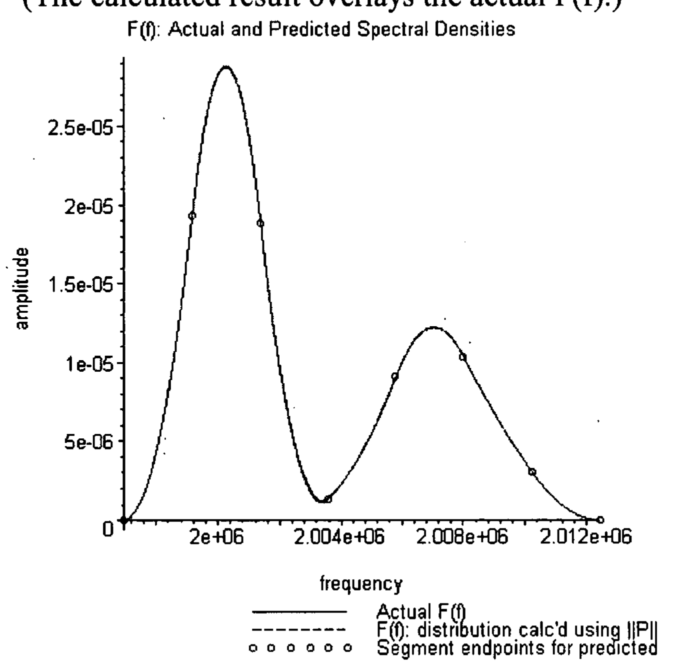

FIG: 24

Equations for Piecewise Continuous Equally Spaced Quadratic Function Segments

The equation for the first *N-2* columns of ||A|| is (51)

$$A_{i,j}=\frac{1}{8}\mathrm{E}(t)\,((-4N+4j+2)\cos(2\pi t(H_0+(N-1)H_\delta))+2\cos(2(H_0+jH_\delta)\pi t)+2(N-j)\cos(2\pi t(H_0+(N-2)H_\delta))-2\cos(2\pi t(H_0+(j-1)H_\delta))$$
$$+2(N-1-j)\cos(2(H_0+NH_\delta)\pi t))\Big/(\pi^3 t^3)$$

The equation for the last *N-2* columns of ||A|| is (52)

$$A_{i,j+N-2}=\frac{1}{8}\mathrm{E}(t)\,($$
$$2\sin(2\pi t(H_0+(j-1)H_\delta))+\sin(2\pi t(H_0+(N-2)H_\delta))(-2N+2j)-2(N-1-j)\sin(2(H_0+NH_\delta)\pi t)-2\sin(2(H_0+jH_\delta)\pi t)+(4N-4j-2)\sin(2\pi t(H_0+(N-1)H_\delta)))$$
$$\Big/(\pi^3 t^3)$$

The equations for calculating the *Nth* and *N-1th* terms of |a| and |x| from the first N-2 terms

$$a_{N-1}=\sum_{p=1}^{N-2}a_p\,(p-N)\quad(43)\qquad a_N=-\left(\sum_{p=1}^{N-2}a_p\,(p-N+1)\right)\quad(44)$$

The equation for spectral density component functions (53)

$$\mathrm{d}(f)=\left(\sum_{j=1}^{N-1}\mathrm{Heaviside}(f-H_0-(j-1)H_\delta)\,\mathrm{Heaviside}(H_0+jH_\delta-f)\left(H_\delta\left(\sum_{p=1}^{j-1}(-(2p-1)H_\delta+2f-2H_0)\,a_p\right)+a_j\,(f-H_0-(j-1)H_\delta)^2\right)\right)$$
$$+\mathrm{Heaviside}(f-H_0-(N-1)H_\delta)\,\mathrm{Heaviside}(H_0-f+NH_\delta)\left(\sum_{j=1}^{N-2}(-a_j\,(H_0-f+NH_\delta)^2\,(j+1-N))\right)$$

The equation for calculating estimated functions of time (50)

$$\mathrm{g}(t)=\mathrm{E}(t)\left(\sum_{j=1}^{N-2}\left(\frac{\frac{1}{8}(\sin(2\pi t(H_0+(N-2)H_\delta))(-2N+2j)-2(N-1-j)\sin(2\pi t(H_0+NH_\delta))-2\sin(2\pi t(H_0+jH_\delta))+(4N-4j-2)\sin(2\pi t(H_0+(N-1)H_\delta))+2\sin(2\pi t(H_0+(j-1)H_\delta)))\,x_j}{\pi^3 t^3}\right.\right.$$
$$\left.\left.+\frac{\frac{1}{8}(2(N-j)\cos(2\pi t(H_0+(N-2)H_\delta))+2\cos(2\pi t(H_0+jH_\delta))+2(N-1-j)\cos(2\pi t(H_0+NH_\delta))+(-4N+4j+2)\cos(2\pi t(H_0+(N-1)H_\delta))-2\cos(2\pi t(H_0+(j-1)H_\delta)))\,a_j}{\pi^3 t^3}\right)\right)$$

$$d(f) = \left(\sum_{j=1}^{N-1} \mathrm{Heaviside}(f - H_0 - (j-1) H_\delta) \, \mathrm{Heaviside}(H_0 + j H_\delta - f) \left(H_\delta \left(\sum_{p=1}^{j-1} (-(2p-1) H_\delta + 2f - 2 H_0) \, a_p \right) + a_j (f - H_0 - (j-1) H_\delta)^2 \right) \right)$$
$$+ \mathrm{Heaviside}(f - H_0 - (N-1) H_\delta) \, \mathrm{Heaviside}(H_0 - f + N H_\delta) \left(\sum_{j=1}^{N-2} (-a_j (H_0 - f + N H_\delta)^2 (j + 1 - N)) \right) \quad (53)$$

FIG: 25

Equations for Piecewise Continuous Equally Spaced Linear Function Segments

The equation for the first *N-1* columns of ||A|| is

$$A_{i,j} = \frac{1}{4}\,\frac{\mathrm{E}(t)\,(\sin(2\,\pi\,t_i\,(H_0 + (N-1)\,H_\delta)) - \sin(2\,(H_0 + N\,H_\delta)\,\pi\,t_i) - \sin(2\,\pi\,t_i\,(H_0 + (j-1)\,H_\delta)) + \sin(2\,\pi\,(H_0 + j\,H_\delta)\,t_i))}{\pi^2\,t_i^2}$$

The equation for the last *N-1* columns of ||A|| is

$$A_{i,j+N-1} = \frac{1}{4}\,\frac{\mathrm{E}(t)\,(\cos(2\,\pi\,t_i\,(H_0 + (N-1)\,H_\delta)) - \cos(2\,(H_0 + N\,H_\delta)\,\pi\,t_i) - \cos(2\,\pi\,t_i\,(H_0 + (j-1)\,H_\delta)) + \cos(2\,\pi\,(H_0 + j\,H_\delta)\,t_i))}{\pi^2\,t_i^2}$$

The equation for calculating the *Nth* term of |a| and |x| from the first N-1 terms

$$a_N = -\left(\sum_{p=1}^{N-1} a_p\right)$$

The equation for spectral density component functions

$$\mathrm{d}(f) = \left(\sum_{j=1}^{N-1} \mathrm{Heaviside}(f - H_0 - j\,H_\delta + H_\delta)\,\mathrm{Heaviside}(H_0 + j\,H_\delta - f)\left(-H_\delta\,a_j\,j + H_\delta\,a_j + H_\delta\left(\sum_{p=1}^{j-1} a_p\right) + a_j\,f - a_j\,H_0\right)\right)$$

$$+\,\mathrm{Heaviside}(f - H_0 - N\,H_\delta + H_\delta)\,\mathrm{Heaviside}(H_0 + N\,H_\delta - f)\left(\sum_{j=1}^{N-1} (-(f - H_0 - N\,H_\delta + H_\delta)\,a_j)\right) + \mathrm{Heaviside}(f - H_0 - N\,H_\delta + H_\delta)\,\mathrm{Heaviside}(H_0 + N\,H_\delta - f)\left(\sum_{p=1}^{N-1} H_\delta\,a_p\right)$$

The equation for calculating estimated functions of time

$$\mathrm{g}(t) = \mathrm{E}(t)\left(\frac{1}{4}\left(\sum_{j=1}^{N} \frac{(\sin(2\,\pi\,t\,(H_0 + (N-1)\,H_\delta)) - \sin(2\,(H_0 + N\,H_\delta)\,\pi\,t) - \sin(2\,\pi\,t\,(H_0 + (j-1)\,H_\delta)) + \sin(2\,\pi\,(H_0 + j\,H_\delta)\,t))\,a_j}{\pi^2\,t^2}\right)\right.$$

$$\left. + \frac{1}{4}\left(\sum_{j=1}^{N} \frac{(\cos(2\,\pi\,t\,(H_0 + (N-1)\,H_\delta)) - \cos(2\,(H_0 + N\,H_\delta)\,\pi\,t) - \cos(2\,\pi\,t\,(H_0 + (j-1)\,H_\delta)) + \cos(2\,\pi\,(H_0 + j\,H_\delta)\,t))\,x_j}{\pi^2\,t^2}\right)\right)$$

FIG. 26

Three Frequency Function with Four Digitization Bins

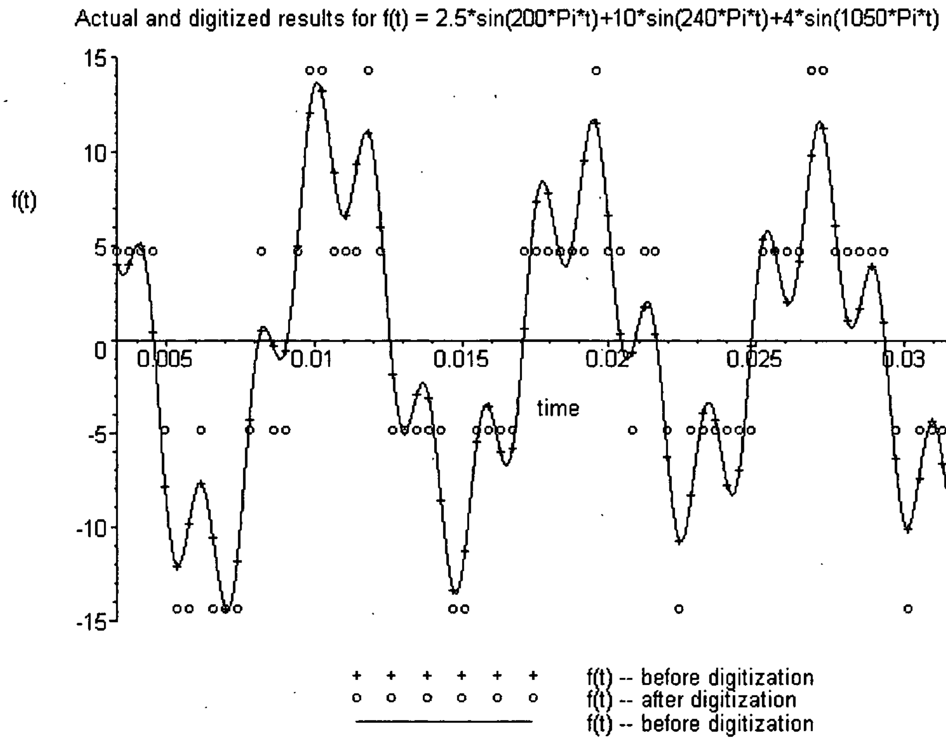

FIG. 27

Three Frequency Function with Four Digitization Bins and Least Squared Errors Estimate

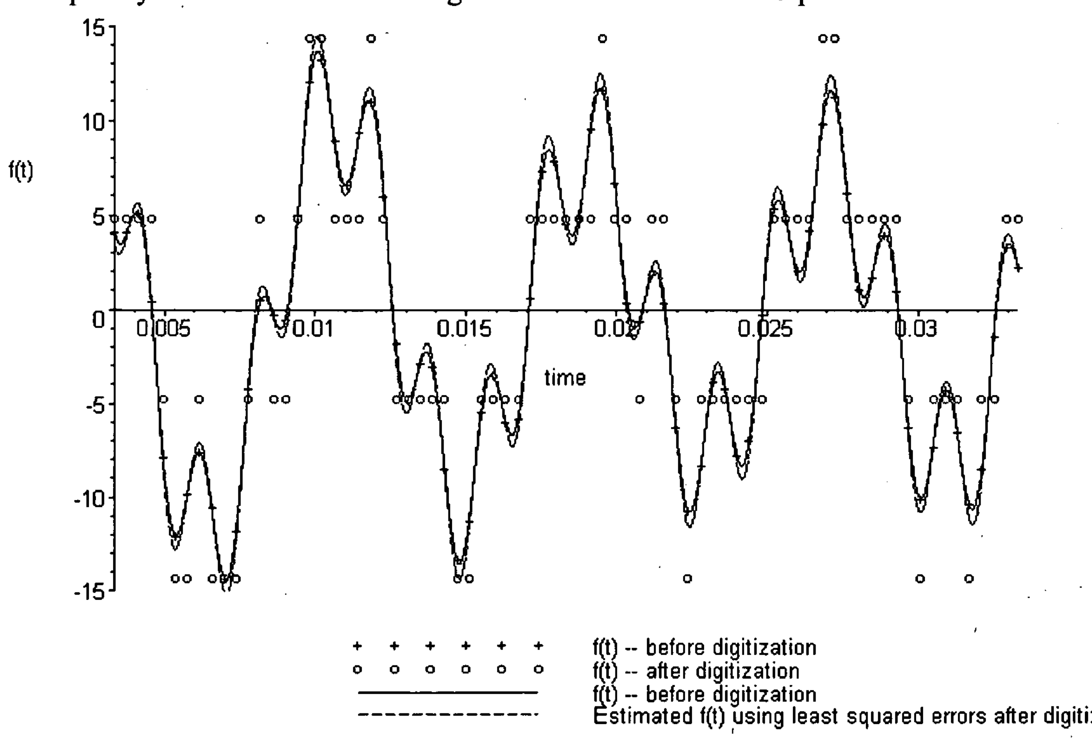

FIG. 28

Three Frequency Function with Four Digitization Bins and Least Absolute Value Errors Estimate

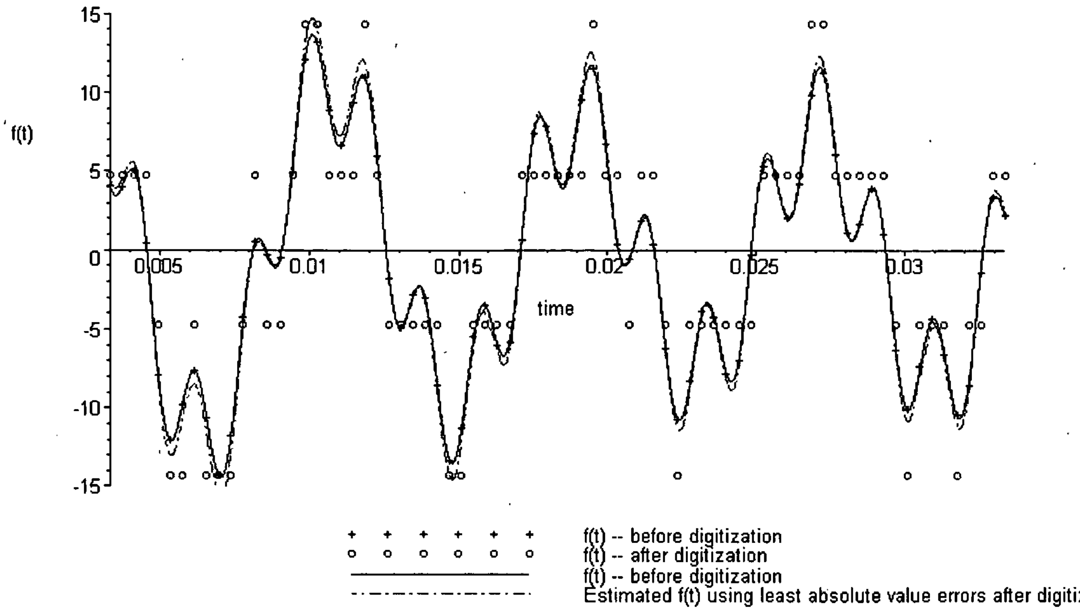

FIG. 29

Three Frequency Function with Four Digitization Bins and Least Squared Errors and Least Absolute Value Errors Estimates

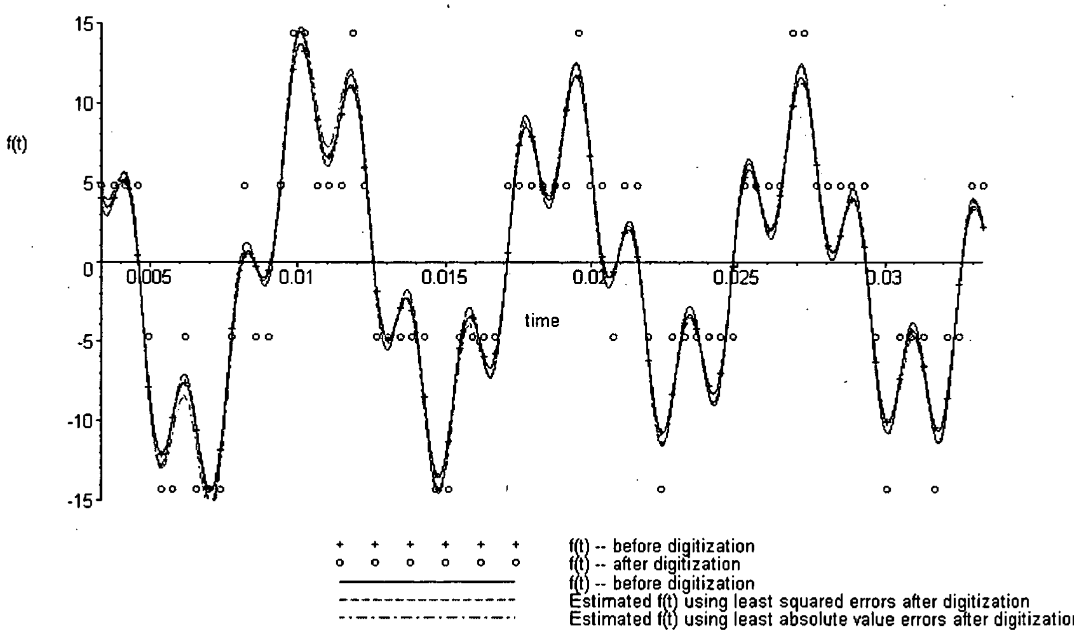

FIG. 30

Actual and Estimated G(f) Calculated without Accounting for Digitization

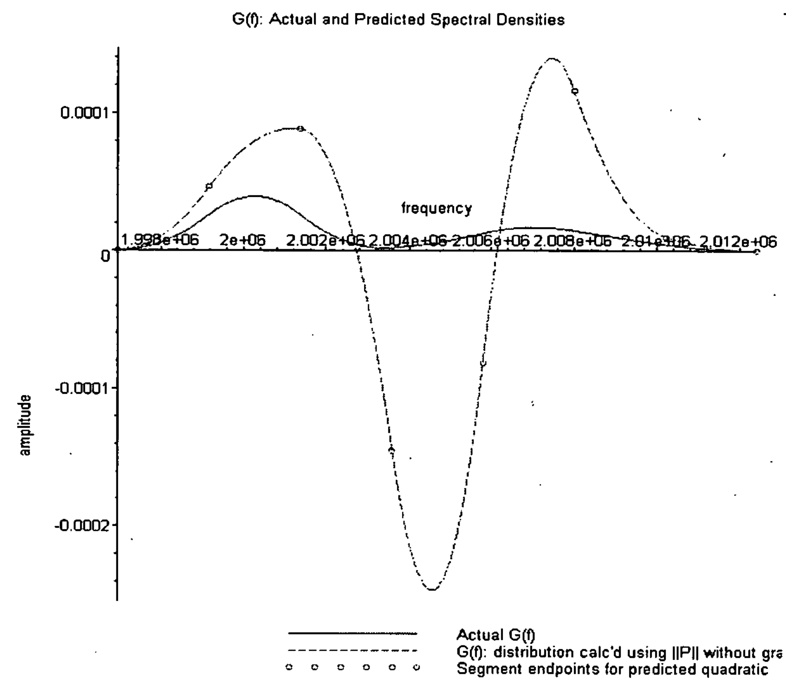

FIG. 31

Actual and Estimated F(f) Calculated without Accounting for Digitization

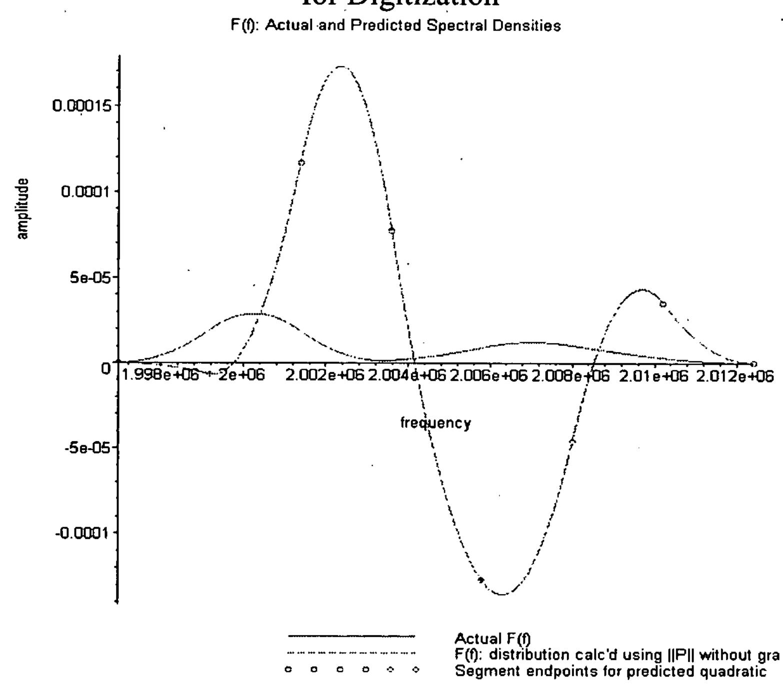

FIG. 32

Bounded Area Measure when Using Piecewise Continuous Quadratic Function Segments

$$MAs_{1..N} = \left(\sum_{j=1}^{N-2} \left(3a_j^2 + 15\left(\sum_{q=1}^{j-1} (2j-2q-1)a_q \right)^2 + 30\left(\sum_{q=1}^{j-1} (2j-2q-1)a_q \right)\left(\sum_{q=1}^{j-1} a_q \right) + 10a_j\left(\sum_{q=1}^{j-1} (2j-2q-1)a_q \right) + 20\left(\sum_{q=1}^{j-1} a_q \right)^2 + 15a_j\left(\sum_{q=1}^{j-1} a_q \right) \right) \right)$$

$$+68\left(\sum_{q=1}^{N-2} a_q(q-N) \right)^2 + 15\left(\sum_{q=1}^{N-2} (3-2N+2q)a_q \right)^2 - 30\left(\sum_{q=1}^{N-2} (3-2N+2q)a_q \right)\left(\sum_{q=1}^{N-2} a_q \right) - 10\left(\sum_{q=1}^{N-2} a_q(q-N) \right)\left(\sum_{q=1}^{N-2} (3-2N+2q)a_q \right) + 40\left(\sum_{q=1}^{N-2} a_q \right)^2$$

$$+85\left(\sum_{q=1}^{N-2} a_q(q-N) \right)\left(\sum_{q=1}^{N-2} a_q \right) + 3\left(\sum_{q=1}^{N-2} a_q(q+1-N) \right)^2 + 15\left(\sum_{q=1}^{N-2} (1-2N+2q)a_q \right)^2 - 60\left(\sum_{q=1}^{N-2} (1-2N+2q)a_q \right)\left(\sum_{q=1}^{N-2} a_q(q-N) \right)$$

$$-30\left(\sum_{q=1}^{N-2} (1-2N+2q)a_q \right)\left(\sum_{q=1}^{N-2} a_q \right) + 10\left(\sum_{q=1}^{N-2} a_q(q+1-N) \right)\left(\sum_{q=1}^{N-2} (1-2N+2q)a_q \right) - 25\left(\sum_{q=1}^{N-2} a_q(q+1-N) \right)\left(\sum_{q=1}^{N-2} a_q(q-N) \right)$$

$$-15\left(\sum_{q=1}^{N-2} a_q(q+1-N) \right)\left(\sum_{q=1}^{N-2} a_q \right)$$

FIG. 33

Arc Length Measures for Spectral Density Component Functions Based on Piecewise Continuous Quadratic Function Segments

$$M_{arc\_length,\,G(f)} = 5\left(\sum_{j=1}^{N-2} a_j\right)\left(\sum_{j=1}^{N-2} a_j(-N+j)\right) + 2\left(\sum_{j=1}^{N-2} a_j(-N+j)\right)^2 + 4\left(\sum_{j=1}^{N-2} a_j\right)^2 + \left(\sum_{j=1}^{N-2}\left(3\left(\sum_{k=1}^{j} a_k\right)\left(\sum_{q=1}^{j-1} a_q\right) + a_j^2\right)\right)$$

$$M_{arc\_length,\,F(f)} = 5\left(\sum_{j=1}^{N-2} x_j\right)\left(\sum_{j=1}^{N-2} x_j(-N+j)\right) + 2\left(\sum_{j=1}^{N-2} x_j(-N+j)\right)^2 + 4\left(\sum_{j=1}^{N-2} x_j\right)^2 + \left(\sum_{j=1}^{N-2}\left(3\left(\sum_{k=1}^{j} x_k\right)\left(\sum_{q=1}^{j-1} x_q\right) + x_j^2\right)\right)$$

FIG. 34

Bounded Area Measures for Spectral Density Component Functions Based on Piecewise Continuous Quadratic Function Segments

$$M_{bounded\_area,\,G(f)} = \left(\sum_{j=1}^{N-2}\left(3a_j^2 + 15\left(\sum_{q=1}^{j-1}(2j-2q-1)a_q\right)^2 + 30\left(\sum_{q=1}^{j-1}(2j-2q-1)a_q\right)\left(\sum_{q=1}^{j-1}a_q\right) + 10a_j\left(\sum_{q=1}^{j-1}(2j-2q-1)a_q\right) + 20\left(\sum_{q=1}^{j-1}a_q\right)^2 + 15a_j\left(\sum_{q=1}^{j-1}a_q\right)\right)\right)$$
$$+68\left(\sum_{q=1}^{N-2}a_q(q-N)\right)^2 + 15\left(\sum_{q=1}^{N-2}(3-2N+2q)a_q\right)^2 - 30\left(\sum_{q=1}^{N-2}(3-2N+2q)a_q\right)\left(\sum_{q=1}^{N-2}a_q\right) - 10\left(\sum_{q=1}^{N-2}a_q(q-N)\right)\left(\sum_{q=1}^{N-2}(3-2N+2q)a_q\right) + 40\left(\sum_{q=1}^{N-2}a_q\right)^2$$
$$+85\left(\sum_{q=1}^{N-2}a_q(q-N)\right)\left(\sum_{q=1}^{N-2}a_q\right) + 3\left(\sum_{q=1}^{N-2}a_q(q+1-N)\right)^2 + 15\left(\sum_{q=1}^{N-2}(1-2N+2q)a_q\right)^2 - 60\left(\sum_{q=1}^{N-2}(1-2N+2q)a_q\right)\left(\sum_{q=1}^{N-2}a_q(q-N)\right)$$
$$-30\left(\sum_{q=1}^{N-2}(1-2N+2q)a_q\right)\left(\sum_{q=1}^{N-2}a_q\right) + 10\left(\sum_{q=1}^{N-2}a_q(q+1-N)\right)\left(\sum_{q=1}^{N-2}(1-2N+2q)a_q\right) - 25\left(\sum_{q=1}^{N-2}a_q(q+1-N)\right)\left(\sum_{q=1}^{N-2}a_q(q-N)\right)$$
$$-15\left(\sum_{q=1}^{N-2}a_q(q+1-N)\right)\left(\sum_{q=1}^{N-2}a_q\right)$$

$$M_{bounded\_area,\,F(f)} = \left(\sum_{j=1}^{N-2}\left(3x_j^2 + 15\left(\sum_{q=1}^{j-1}(2j-2q-1)x_q\right)^2 + 30\left(\sum_{q=1}^{j-1}(2j-2q-1)x_q\right)\left(\sum_{q=1}^{j-1}x_q\right) + 10x_j\left(\sum_{q=1}^{j-1}(2j-2q-1)x_q\right) + 20\left(\sum_{q=1}^{j-1}x_q\right)^2 + 15x_j\left(\sum_{q=1}^{j-1}x_q\right)\right)\right)$$
$$+68\left(\sum_{q=1}^{N-2}x_q(q-N)\right)^2 + 15\left(\sum_{q=1}^{N-2}(3-2N+2q)x_q\right)^2 - 30\left(\sum_{q=1}^{N-2}(3-2N+2q)x_q\right)\left(\sum_{q=1}^{N-2}x_q\right) - 10\left(\sum_{q=1}^{N-2}x_q(q-N)\right)\left(\sum_{q=1}^{N-2}(3-2N+2q)x_q\right) + 40\left(\sum_{q=1}^{N-2}x_q\right)^2$$
$$+85\left(\sum_{q=1}^{N-2}x_q(q-N)\right)\left(\sum_{q=1}^{N-2}x_q\right) + 3\left(\sum_{q=1}^{N-2}x_q(q+1-N)\right)^2 + 15\left(\sum_{q=1}^{N-2}(1-2N+2q)x_q\right)^2 - 60\left(\sum_{q=1}^{N-2}(1-2N+2q)x_q\right)\left(\sum_{q=1}^{N-2}x_q(q-N)\right)$$
$$-30\left(\sum_{q=1}^{N-2}(1-2N+2q)x_q\right)\left(\sum_{q=1}^{N-2}x_q\right) + 10\left(\sum_{q=1}^{N-2}x_q(q+1-N)\right)\left(\sum_{q=1}^{N-2}(1-2N+2q)x_q\right) - 25\left(\sum_{q=1}^{N-2}x_q(q+1-N)\right)\left(\sum_{q=1}^{N-2}x_q(q-N)\right)$$
$$-15\left(\sum_{q=1}^{N-2}x_q(q+1-N)\right)\left(\sum_{q=1}^{N-2}x_q\right)$$

FIG. 35

Quadratic Curvature Measures for Spectral Density Component Functions Based on Piecewise Continuous Quadratic Function Segments

$$M_{curvature\,,\,\mathrm{G}(f)} = \left(\sum_{j=1}^{N-2} a_j^{\,2} \right) + \left(\sum_{p=1}^{N-2} a_p \,(p - N) \right)^2 + \left(\sum_{p=1}^{N-2} a_p \,(p + 1 - N) \right)^2$$

$$M_{curvature\,,\,\mathrm{F}(f)} = \left(\sum_{j=1}^{N-2} x_j^{\,2} \right) + \left(\sum_{p=1}^{N-2} x_p \,(p - N) \right)^2 + \left(\sum_{p=1}^{N-2} x_p \,(p + 1 - N) \right)^2$$

FIG. 36

Objective Function Combining Measure of Arc Length and Measure of Area for both G(f) and F(f)

$$
\begin{aligned}
ObjF = {} & 15\left(\sum_{q=1}^{N-2}(1-2N+2q)a_q\right)^2 \\
& +\left(\sum_{j=1}^{N-2}\left(3a_j^2+15\left(\sum_{q=1}^{j-1}(2j-2q-1)a_q\right)^2+30\left(\sum_{q=1}^{j-1}(2j-2q-1)a_q\right)\left(\sum_{q=1}^{j-1}a_q\right)+10a_j\left(\sum_{q=1}^{j-1}(2j-2q-1)a_q\right)+20\left(\sum_{q=1}^{j-1}a_q\right)^2+15a_j\left(\sum_{q=1}^{j-1}a_q\right)\right)\right) \\
& +68\left(\sum_{q=1}^{N-2}a_q(q-N)\right)^2+15\left(\sum_{q=1}^{N-2}(3-2N+2q)a_q\right)^2+40\left(\sum_{q=1}^{N-2}a_q\right)^2+3\left(\sum_{q=1}^{N-2}a_q(q+1-N)\right)^2-30\left(\sum_{q=1}^{N-2}(3-2N+2q)a_q\right)\left(\sum_{q=1}^{N-2}a_q\right) \\
& -10\left(\sum_{q=1}^{N-2}a_q(q-N)\right)\left(\sum_{q=1}^{N-2}(3-2N+2q)a_q\right)+85\left(\sum_{q=1}^{N-2}a_q(q-N)\right)\left(\sum_{q=1}^{N-2}a_q\right)-60\left(\sum_{q=1}^{N-2}(1-2N+2q)a_q\right)\left(\sum_{q=1}^{N-2}a_q(q-N)\right)-30\left(\sum_{q=1}^{N-2}(1-2N+2q)a_q\right)\left(\sum_{q=1}^{N-2}a_q\right) \\
& +10\left(\sum_{q=1}^{N-2}a_q(q+1-N)\right)\left(\sum_{q=1}^{N-2}(1-2N+2q)a_q\right)-25\left(\sum_{q=1}^{N-2}a_q(q+1-N)\right)\left(\sum_{q=1}^{N-2}a_q(q-N)\right)-15\left(\sum_{q=1}^{N-2}a_q(q+1-N)\right)\left(\sum_{q=1}^{N-2}a_q\right)+2\left(\sum_{j=1}^{N-2}a_j(-N+j)\right)^2+4\left(\sum_{j=1}^{N-2}a_j\right)^2 \\
& +\left(\sum_{j=1}^{N-2}\left(3\left(\sum_{k=1}^{j}a_k\right)\left(\sum_{q=1}^{j-1}a_q\right)+a_j^2\right)\right)+5\left(\sum_{j=1}^{N-2}a_j\right)\left(\sum_{j=1}^{N-2}a_j(-N+j)\right)+2\left(\sum_{j=1}^{N-2}x_j(-N+j)\right)^2+4\left(\sum_{j=1}^{N-2}x_j\right)^2+\left(\sum_{j=1}^{N-2}\left(3\left(\sum_{k=1}^{j}x_k\right)\left(\sum_{q=1}^{j-1}x_q\right)+x_j^2\right)\right) \\
& +5\left(\sum_{j=1}^{N-2}x_j\right)\left(\sum_{j=1}^{N-2}x_j(-N+j)\right)-30\left(\sum_{q=1}^{N-2}(3-2N+2q)x_q\right)\left(\sum_{q=1}^{N-2}x_q\right)-10\left(\sum_{q=1}^{N-2}x_q(q-N)\right)\left(\sum_{q=1}^{N-2}(3-2N+2q)x_q\right)+85\left(\sum_{q=1}^{N-2}x_q(q-N)\right)\left(\sum_{q=1}^{N-2}x_q\right) \\
& -60\left(\sum_{q=1}^{N-2}(1-2N+2q)x_q\right)\left(\sum_{q=1}^{N-2}x_q(q-N)\right)-30\left(\sum_{q=1}^{N-2}(1-2N+2q)x_q\right)\left(\sum_{q=1}^{N-2}x_q\right)+10\left(\sum_{q=1}^{N-2}x_q(q+1-N)\right)\left(\sum_{q=1}^{N-2}(1-2N+2q)x_q\right) \\
& -25\left(\sum_{q=1}^{N-2}x_q(q+1-N)\right)\left(\sum_{q=1}^{N-2}x_q(q-N)\right)-15\left(\sum_{q=1}^{N-2}x_q(q+1-N)\right)\left(\sum_{q=1}^{N-2}x_q\right) \\
& +\left(\sum_{j=1}^{N-2}\left(3x_j^2+15\left(\sum_{q=1}^{j-1}(2j-2q-1)x_q\right)^2+30\left(\sum_{q=1}^{j-1}(2j-2q-1)x_q\right)\left(\sum_{q=1}^{j-1}x_q\right)+10x_j\left(\sum_{q=1}^{j-1}(2j-2q-1)x_q\right)+20\left(\sum_{q=1}^{j-1}x_q\right)^2+15x_j\left(\sum_{q=1}^{j-1}x_q\right)\right)\right) \\
& +68\left(\sum_{q=1}^{N-2}x_q(q-N)\right)^2+15\left(\sum_{q=1}^{N-2}(3-2N+2q)x_q\right)^2+40\left(\sum_{q=1}^{N-2}x_q\right)^2+3\left(\sum_{q=1}^{N-2}x_q(q+1-N)\right)^2+15\left(\sum_{q=1}^{N-2}(1-2N+2q)x_q\right)^2
\end{aligned}
$$

FIG. 37

Double Sided Constraint Violation Value Function

(Digitization bin width = 1)

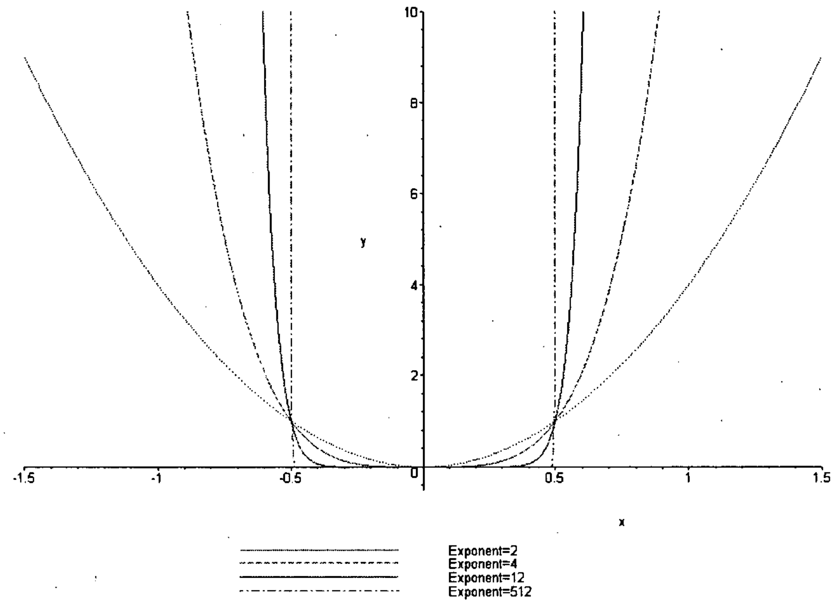

FIG. 38

Nonnegativity Constraint Value Function 1 (NCV1)

$$
\begin{aligned}
NCV1 = p^{r}\Bigg(&\sum_{j=2}^{N-2}\frac{1}{a_j^{2}\mu}\left(-a_j\left(\sum_{q=1}^{j-1}a_q(-2q-1+2j)\right)+\left(\sum_{q=1}^{j-1}a_q\right)^{2}\right)^{2}\left(-\frac{1}{2}\frac{\sum_{q=1}^{j-1}a_q}{a_j\sqrt{\frac{\left(\sum_{q=1}^{j-1}a_q\right)^{2}}{a_j^{2}}+k^{p}}}+\frac{1}{2}\right)\left(\frac{1}{2}\frac{1+\frac{\sum_{q=1}^{j-1}a_q}{a_j}}{\sqrt{\left(1+\frac{\sum_{q=1}^{j-1}a_q}{a_j}\right)^{2}+k^{p}}}+\frac{1}{2}\right)\\
&\cdot\left(\frac{1}{2}\frac{-a_j\left(\sum_{q=1}^{j-1}a_q(-2q-1+2j)\right)+\left(\sum_{q=1}^{j-1}a_q\right)^{2}}{a_j\sqrt{\frac{\left(-a_j\left(\sum_{q=1}^{j-1}a_q(-2q-1+2j)\right)+\left(\sum_{q=1}^{j-1}a_q\right)^{2}\right)^{2}}{a_j^{2}}+\mu k^{p}}}+\frac{1}{2}\right)\left(\frac{1}{2}\frac{a_j}{\sqrt{a_j^{2}+\mu k^{p}}}+\frac{1}{2}\right)+\\
&\left(-\left(\sum_{q=1}^{N-2}a_q(q-N)\right)\left(\sum_{q=1}^{N-2}a_q(-2q-3+2N)\right)+\left(\sum_{q=1}^{N-2}a_q\right)^{2}\right)^{2}\left(-\frac{1}{2}\frac{\sum_{q=1}^{N-2}a_q}{\left(\sum_{q=1}^{N-2}a_q(q-N)\right)\sqrt{\frac{\left(\sum_{q=1}^{N-2}a_q\right)^{2}}{\left(\sum_{q=1}^{N-2}a_q(q-N)\right)^{2}}+k^{p}}}+\frac{1}{2}\right)\left(\frac{1}{2}\frac{1+\frac{\sum_{q=1}^{N-2}a_q}{\sum_{q=1}^{N-2}a_q(q-N)}}{\sqrt{\left(1+\frac{\sum_{q=1}^{N-2}a_q}{\sum_{q=1}^{N-2}a_q(q-N)}\right)^{2}+k^{p}}}+\frac{1}{2}\right)\\
&\cdot\left(\frac{1}{2}\frac{-\left(\sum_{q=1}^{N-2}a_q(q-N)\right)\left(\sum_{q=1}^{N-2}a_q(-2q-3+2N)\right)+\left(\sum_{q=1}^{N-2}a_q\right)^{2}}{\left(\sum_{q=1}^{N-2}a_q(q-N)\right)\sqrt{\frac{\left(-\left(\sum_{q=1}^{N-2}a_q(q-N)\right)\left(\sum_{q=1}^{N-2}a_q(-2q-3+2N)\right)+\left(\sum_{q=1}^{N-2}a_q\right)^{2}\right)^{2}}{\left(\sum_{q=1}^{N-2}a_q(q-N)\right)^{2}}+\mu k^{p}}}+\frac{1}{2}\right)\\
&\cdot\left(\frac{1}{2}\frac{\sum_{q=1}^{N-2}a_q(q-N)}{\sqrt{\left(\sum_{q=1}^{N-2}a_q(q-N)\right)^{2}+\mu k^{p}}}+\frac{1}{2}\right)\Bigg/\left(\left(\sum_{q=1}^{N-2}a_q(q-N)\right)^{2}\mu\right)\Bigg)
\end{aligned}
$$

Note: Select value of r so that $p^{r}=N^{2}$

FIG. 39

Nonnegativity Constraint Value Function 2 (NCV2)

$$
\begin{aligned}
NCV2 = p^{r}\Bigg(&\sum_{j=2}^{N-2} \frac{\left(-a_j\left(\sum_{q=1}^{j-1} a_q(-2q-1+2j)\right)+\left(\sum_{q=1}^{j-1} a_q\right)^2\right)\left(-\frac{1}{2}\frac{\sum_{q=1}^{j-1} a_q}{a_j\sqrt{\frac{\left(\sum_{q=1}^{j-1} a_q\right)^2}{a_j^2}+k^p}}+\frac{1}{2}\right)\left(\frac{1}{2}\frac{1+\frac{\sum_{q=1}^{j-1} a_q}{a_j}}{\sqrt{\left(1+\frac{\sum_{q=1}^{j-1} a_q}{a_j}\right)^2+k^p}}+\frac{1}{2}\right)}{a_j\sqrt{\mu}} \\
&\quad \times \frac{\left(\frac{1}{2}\frac{-a_j\left(\sum_{q=1}^{j-1} a_q(-2q-1+2j)\right)+\left(\sum_{q=1}^{j-1} a_q\right)^2}{a_j\sqrt{\frac{\left(-a_j\left(\sum_{q=1}^{j-1} a_q(-2q-1+2j)\right)+\left(\sum_{q=1}^{j-1} a_q\right)^2\right)^2}{a_j^2}+\mu k^p}}+\frac{1}{2}\right)\left(\frac{1}{2}\frac{a_j}{\sqrt{a_j^2+\mu k^p}}+\frac{1}{2}\right)}{1} + \\
&\left(-\left(\sum_{q=1}^{N-2} a_q(q-N)\right)\left(\sum_{q=1}^{N-2} a_q(-2q-3+2N)\right)+\left(\sum_{q=1}^{N-2} a_q\right)^2\right)\left(-\frac{1}{2}\frac{\sum_{q=1}^{N-2} a_q}{\left(\sum_{q=1}^{N-2} a_q(q-N)\right)\sqrt{\frac{\left(\sum_{q=1}^{N-2} a_q\right)^2}{\left(\sum_{q=1}^{N-2} a_q(q-N)\right)^2}+k^p}}+\frac{1}{2}\right) \\
&\left(\frac{1}{2}\frac{1+\frac{\sum_{q=1}^{N-2} a_q}{\sum_{q=1}^{N-2} a_q(q-N)}}{\sqrt{\left(1+\frac{\sum_{q=1}^{N-2} a_q}{\sum_{q=1}^{N-2} a_q(q-N)}\right)^2+k^p}}+\frac{1}{2}\right) \\
&\left(\frac{1}{2}\frac{-\left(\sum_{q=1}^{N-2} a_q(q-N)\right)\left(\sum_{q=1}^{N-2} a_q(-2q-3+2N)\right)+\left(\sum_{q=1}^{N-2} a_q\right)^2}{\left(\sum_{q=1}^{N-2} a_q(q-N)\right)\sqrt{\frac{\left(-\left(\sum_{q=1}^{N-2} a_q(q-N)\right)\left(\sum_{q=1}^{N-2} a_q(-2q-3+2N)\right)+\left(\sum_{q=1}^{N-2} a_q\right)^2\right)^2}{\left(\sum_{q=1}^{N-2} a_q(q-N)\right)^2}+\mu k^p}}+\frac{1}{2}\right) \\
&\left(\frac{1}{2}\frac{\sum_{q=1}^{N-2} a_q(q-N)}{\sqrt{\left(\sum_{q=1}^{N-2} a_q(q-N)\right)^2+\mu k^p}}+\frac{1}{2}\right) \Big/ \left(\left(\sum_{q=1}^{N-2} a_q(q-N)\right)\sqrt{\mu}\right)\Bigg)
\end{aligned}
$$

Note: When use NCV2 set exponent r=0

FIG. 40
Tapered Heaviside Gate Function Examples

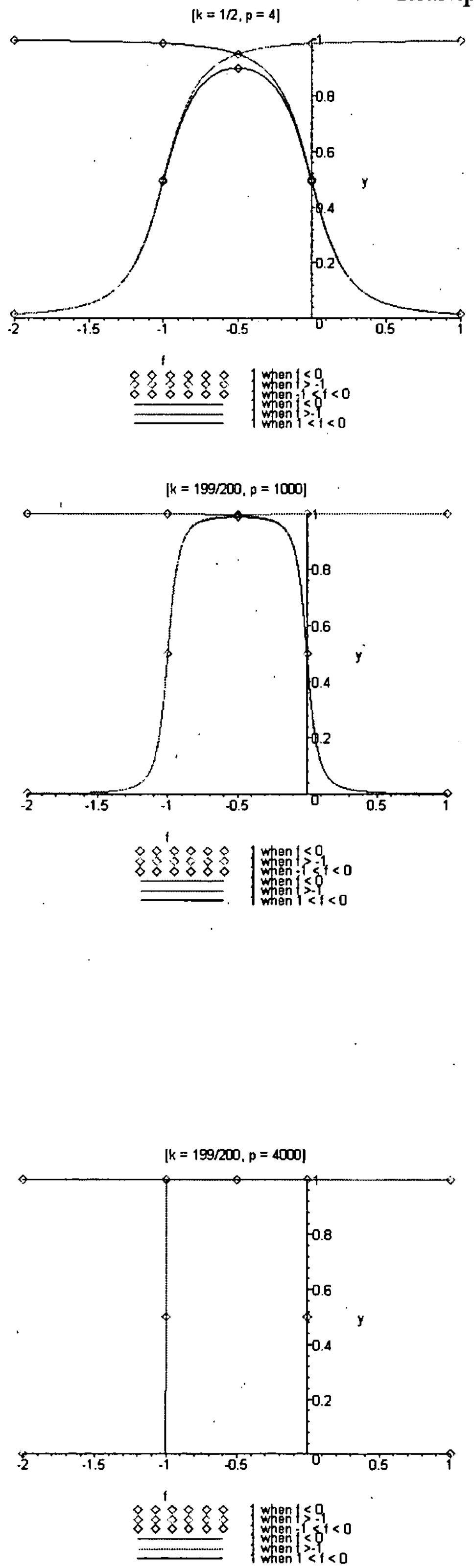

FIG. 41

Tapered Heaviside Gate Function for Extreme Value of G(f) is Less than Zero

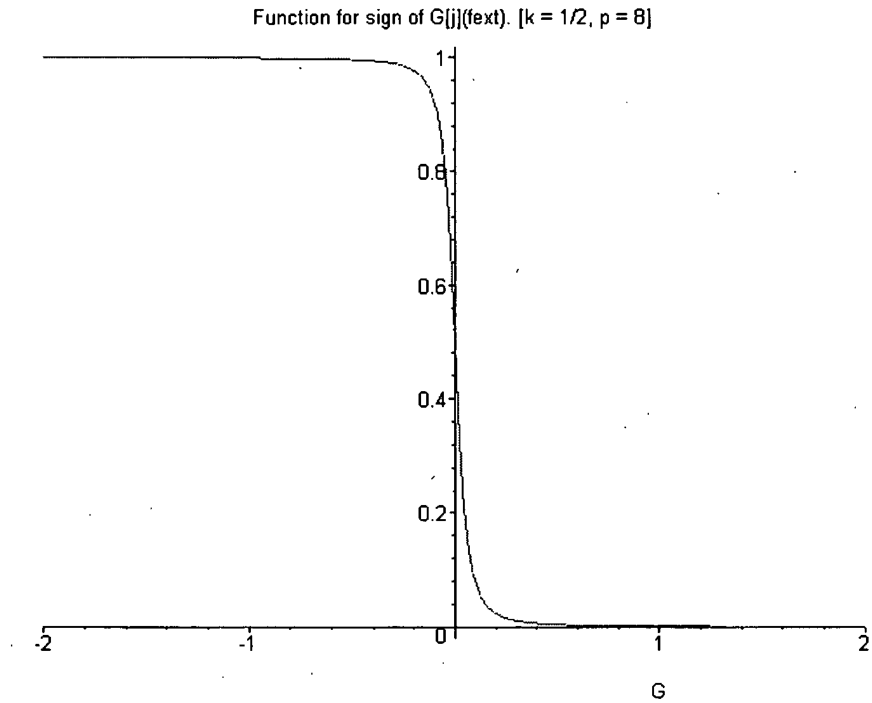

FIG. 42

Example G(f) Calculated Using NCV1 Nonnegativity Constraint Value Function

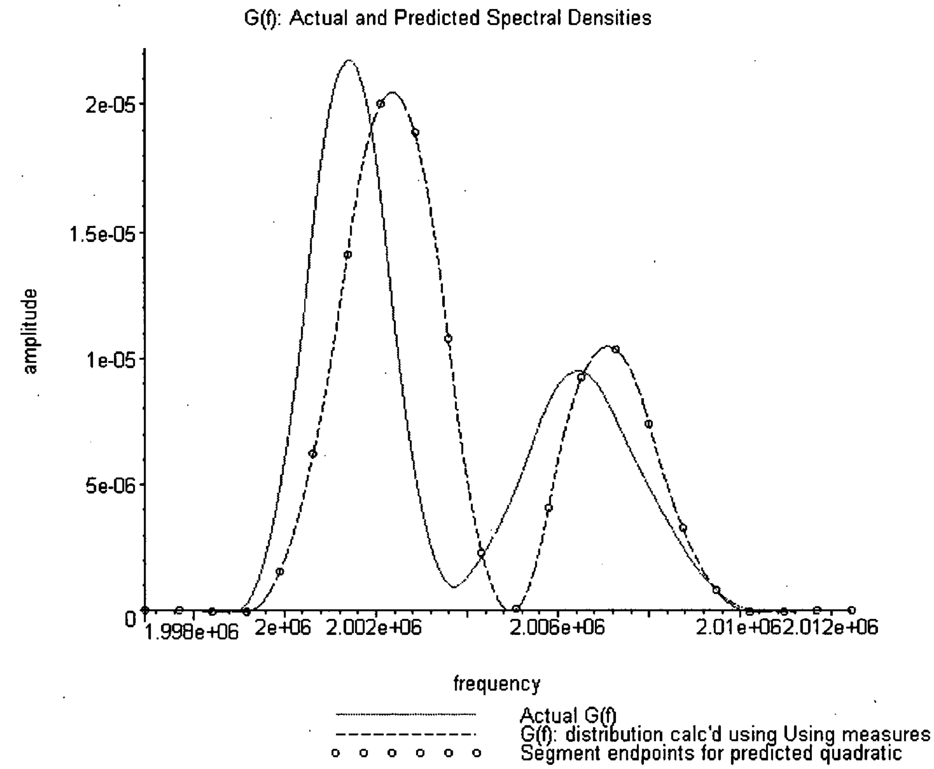

FIG. 43

Peak Count Constraint Value (PCV) Function

$$PCV = \left(H - \left(\sum_{j=2}^{N-2} \left(- \frac{1\, a_j}{2\, \mu^{\left(\frac{1}{2}\right)} \left(\frac{a_j^2}{\mu} + k^{(4p)} \right)^{\left(\frac{1}{2}\right)}} + \frac{1}{2} \right) \left(- \frac{1 \left(\sum_{q=1}^{j-1} a_q \right)}{2\, a_j \left(\frac{\left(\sum_{q=1}^{j-1} a_q \right)^2}{a_j^2} + k^{(4p)} \right)^{\left(\frac{1}{2}\right)}} + \frac{1}{2} \right) \left(\frac{1 \left(1 + \frac{\sum_{q=1}^{j-1} a_q}{a_j} \right)}{2 \left(\left(1 + \frac{\sum_{q=1}^{j-1} a_q}{a_j} \right)^2 + k^{(4p)} \right)^{\left(\frac{1}{2}\right)}} + \frac{1}{2} \right) \right) \right.$$

$$\left. - \left(- \frac{1 \left(\sum_{p=1}^{N-2} a_p (p-N) \right)}{2\, \mu^{\left(\frac{1}{2}\right)} \left(\frac{\left(\sum_{p=1}^{N-2} a_p (p-N) \right)^2}{\mu} + k^{(4p)} \right)^{\left(\frac{1}{2}\right)}} + \frac{1}{2} \right) \left(- \frac{1 \left(\sum_{q=1}^{N-2} a_q \right)}{2 \left(\sum_{p=1}^{N-2} a_p (p-N) \right) \left(\frac{\left(\sum_{q=1}^{N-2} a_q \right)^2}{\left(\sum_{p=1}^{N-2} a_p (p-N) \right)^2} + k^{(4p)} \right)^{\left(\frac{1}{2}\right)}} + \frac{1}{2} \right) \left(\frac{1 \left(1 + \frac{\sum_{q=1}^{N-2} a_q}{\sum_{p=1}^{N-2} a_p (p-N)} \right)}{2 \left(\left(1 + \frac{\sum_{q=1}^{N-2} a_q}{\sum_{p=1}^{N-2} a_p (p-N)} \right)^2 + k^{(4p)} \right)^{\left(\frac{1}{2}\right)}} + \frac{1}{2} \right) \right)^2$$

FIG. 44 Basic Calculation Sequence
Page 1 of 3

1. Specify solution parameters
    A. Specify measurements
        a. Number of measurements (M)
        b. Time interval between measurements
    B. Specify basis frequencies
        a. Lower and upper frequency limits (f_L, f_H)
        b. Number of frequency intervals (N)
    C. Select Objective Function components (refer to FIGS. 33, 34, 35)
        a. Arc length measure for G(f), F(f)
        b. Bounded area measure for G(f), F(f)
        c. Quadratic curvature for G(f), F(f)
    D. Select type of nonnegativity constraint (refer to FIG. 38 and FIG. 39)
        a. Scaled by $G_j(f_{j,ext})^2$
        b. Scaled by $-G_j(f_{j,ext})$ Select peak count constraint (refer to FIG. 43)
        a. Select whether will use peak count constraints
        b. Select number of peaks for G(f), F(f)
    F. Specify properties of digitization bin constraints
        a. Sharpness of constraint penalty (final value of parameter w)
        b. Value of penalty during final calculations (final value of parameter p)
    G. Specify rate of change of nonnegativity constraints (parameter k)

. . . proceed to precalculations

FIG. 45 Basic Calculation Sequence

2. Precalculations (before search for solution)
   A. Calculate matrix ||A|| (Refer to FIG. 25)
   B. Calculate objective function (OF) expression and its Hessian (this will be a matrix of numbers)
   C. Calculate nonnegativity constraint value (NCV) function and its gradient and Hessian (these will be a vector and a matrix of expressions in terms of |a| and |x|).
   D. Calculate peak count constraint value (PCV) function and its gradient and Hessian (these will be a vector and a matrix of expressions in terms of |a| and |x|).
3. Initialize
   A. Select starting |a| and |x| – e.g., constant vectors with small values
   B. Create starting |d| by stacking |a| over |x|
   C. Calculate OF using starting |d| and calculate starting OF scale factor λ (λ=10^(order of magnitude of OF minus 2)
   D. Select starting value of parameter p, e.g., p=2.
   E. Calculate starting values for constraint value functions: digitization bin constraint violation value function (CVV), nonnegativity constraint value function (NCV), peak count constraint value function (PCV)
   F. Calculate value for constrained objective function (COF): COF= OF/λ + CVV + NCV + PCV

. . . proceed to multidimensional Newton method search.

FIG. 46 Basic Calculation Sequence

4. Multidimensional Newton Method Search (designate starting value of variables with subscript k, e.g., starting |d| as $|d_k|$.)
   A. Using current value of p
      a. (This sequence calculates new |d|)
         Using $|d_k|$
         (1) Calculate gradient for OF
         (2) Calculate gradient and Hessian for CVV.
         (3) Calculate gradient and Hessian for NCV.
         (4) Calculate gradient and Hessian for PCV.
         (5) Sum gradients and sum Hessians and use these to calculate the change in |d|: |s|.
         (6) Calculate $|d_{k+1}|=|d_k| + |s|$
         (7) Calculate new OF, CVV, NCV, PCV and COF_{k+1}.
      b. (Backstep, if necessary)
         (1) If $COF_{k+1}<COF_k$ then goto step c.
         (2) Reduce |s| by order of magnitude
         (3) Calculate $|d_{k+1}|=|d_k| + |s|$
         (4) If $COF_{k+1}<COF_k$ then goto step c, otherwise got to step b.(2) and reduce |s|, unless |s| has been reduced to a very small value, in which case goto to step d.
      c. (Get here when have lower COF) If significant change in $|d_{k+1}|$ from $|d_k|$ then
         (1) set $|d_k|=|d_{k+1}|$
         (2) go back to step 4.A.a
         otherwise go to step d.
      d. (Get here when stopped getting changes in $|d_{k+1}|$)
         If at final value of parameter p then go to step B, otherwise
         (1) Increase p, e.g., multiply current value of p by 3/2
         (2) Calculate OF using $|d_k|$ and the new p
         (3) Calculate new value for scale factor λ.
         (4) Go back to step 4.A.
   B. (Get here when have finished calculations with final value of p).

** Search complete **

FIG. 47
Actual and Calculated G(f) from Digitized Data
M=50

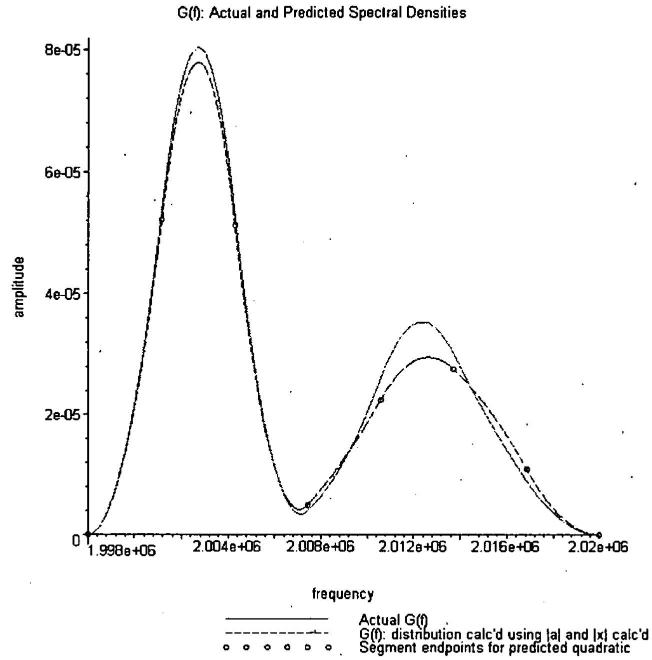

FIG. 48
Actual and Calculated F(f) from Digitized Data
M=50

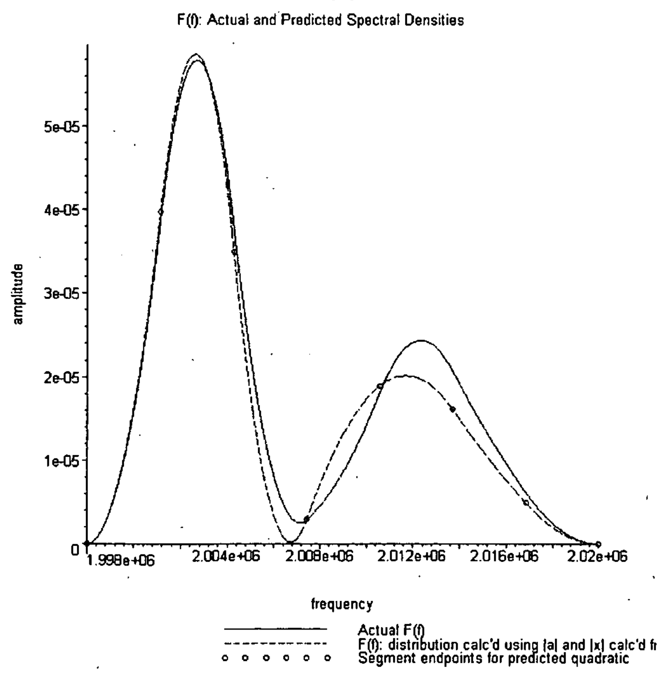

FIG. 49

Actual and Calculated Spectral Density Function from Digitized Data

M=50

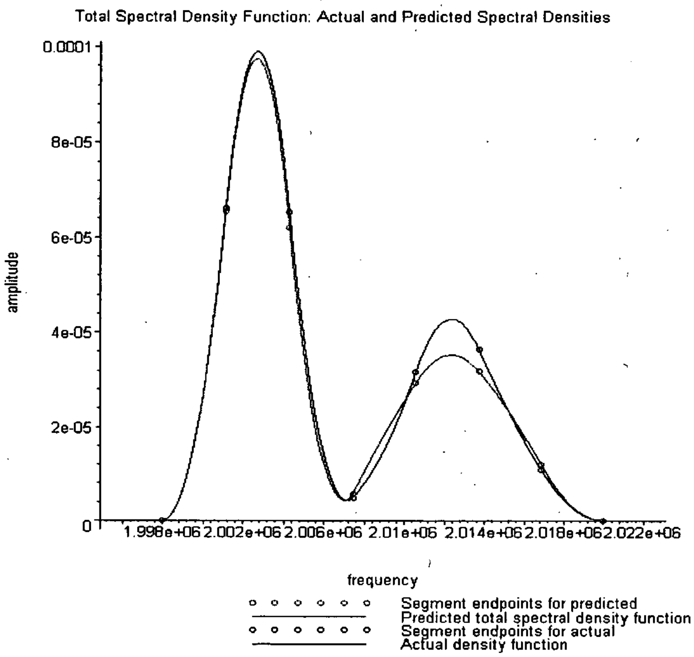

FIG. 50
Actual and Calculated G(f) from Digitized Data
M=200

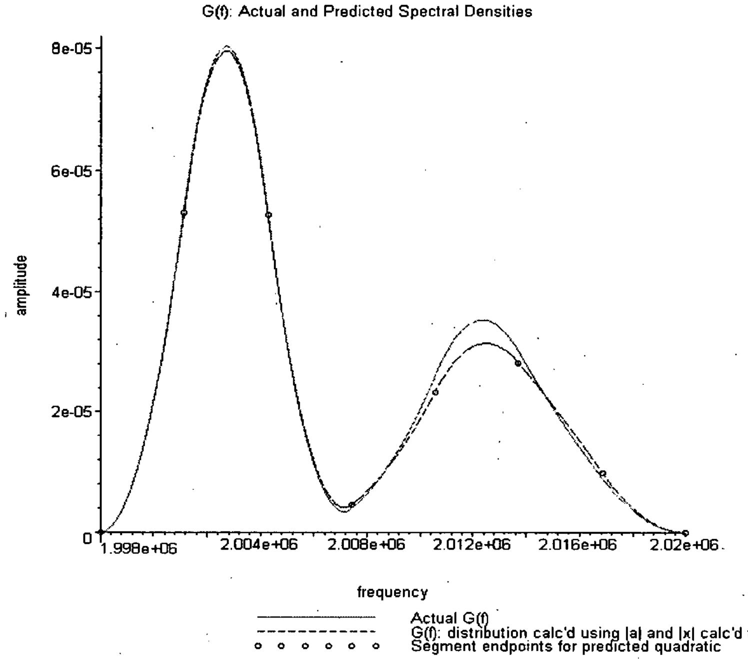

FIG. 51
Actual and Calculated F(f) from Digitized Data
M=200

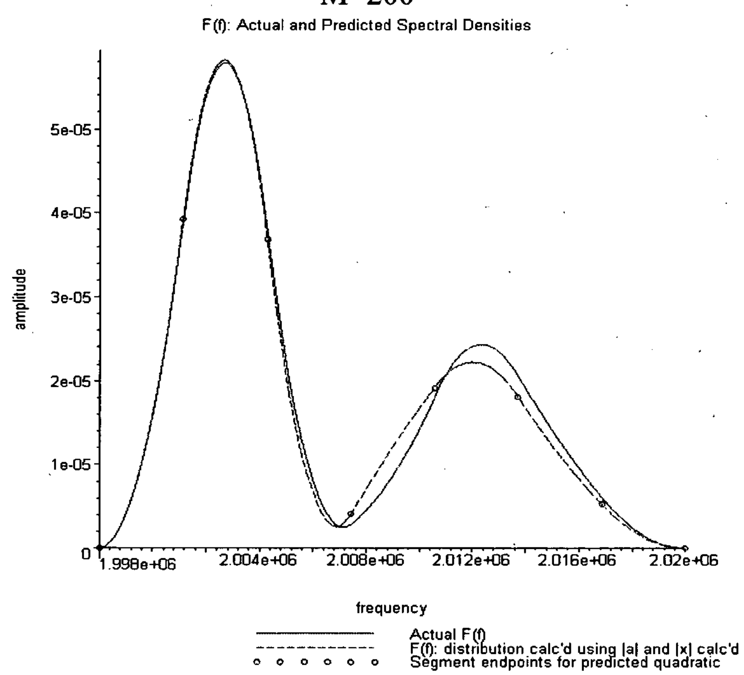

FIG.52
Actual and Calculated Spectral Density Function from Digitized Data
M=200

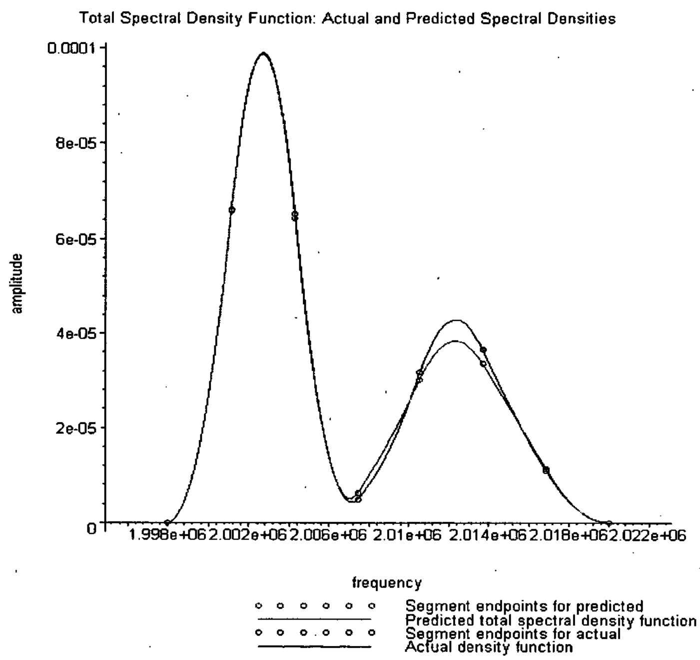

FIG. 53
Actual and Calculated Cumulative Spectral Density Function from Digitized Data
M=200

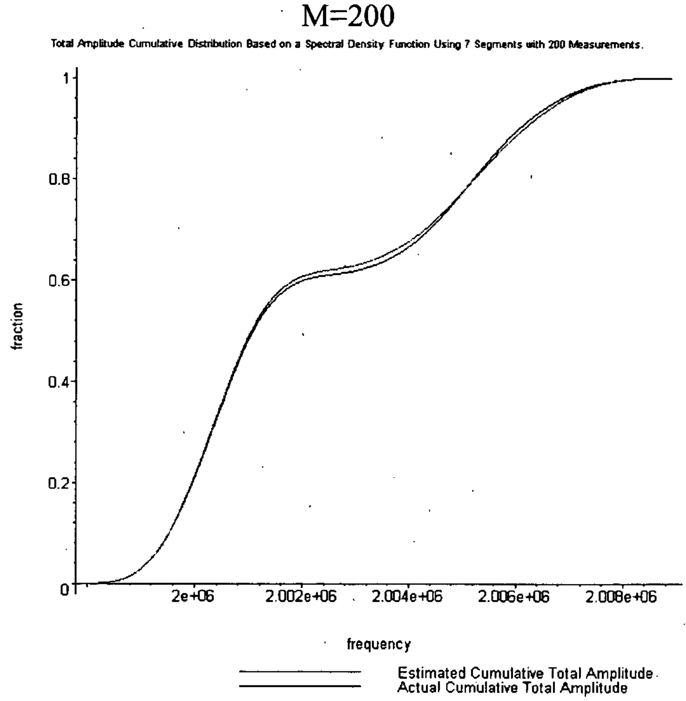

Figure 54